The Derivative

Derivatives in Maple can be done using any one of three commands: diff, Diff or D. Personally I mainly use the diff or Diff commands. Like Limit and limit the uppercase command returns a pretty-print version of the calculation and the lowercase command returns the calculated value.

Using the diff command you simply give the command the function to differentiate and the variable to differentiate with respect to. For example, to differentiate

\[ f(x) = x^3 \sin(x) \]

we execute the following command.

\[ > \text{diff}(x^3\sin(x), x); \]

\[ 3x^2 \sin(x) + x^3 \cos(x) \]

Similarly, we could define the function first and then differentiate the function. For example,

\[ > f := x -> x^3 \sin(x); \]

\[ f := x \rightarrow x^3 \sin(x) \]

\[ > \text{diff}(f(x), x); \]

\[ 3x^2 \sin(x) + x^3 \cos(x) \]

One thing to note here is that the result from the diff command is an expression and not a function. So we cannot substitute values into the derivative expression as it stands. Maple does have a command that will convert an expression to a function. This is the unapply command. For example, the command

\[ > \text{fp} := \text{unapply}(%, x); \]

\[ \text{fp} := x \rightarrow 3x^2 \sin(x) + x^3 \cos(x) \]

will define the function fp as the derivative. Now the statements

\[ > \text{fp}(2); \]

\[ 12 \sin(2) + 8 \cos(2) \]

\[ > \text{evalf}(\%); \]

\[ 7.582394428 \]

will evaluate the derivative at \( x = 2 \).
The Diff command will return a pretty-print version of the derivative. For example, to display
\[ \frac{d}{dx}\left(e^{x^3}\cos\left(\frac{x}{4} + \frac{1}{2}\right)\right) \]
we use,

\[ \text{> Diff}(\text{exp}(x^3)\cos\left((x+2)/4\right),x); \]

To get the derivative, you need to follow this command with the \texttt{value} command. That is,

\[ \text{> value}(\%); \]

Of course, you could define the function first and then take its derivative. For example,

\[ \text{> g:=x->exp(x^3)\cos\left((x+2)/4\right);} \]

\[ g := x \rightarrow e^{x^3}\cos\left(\frac{x}{4} + \frac{1}{2}\right) \]

\[ \text{> Diff}(g(x),x); \]

\[ \frac{d}{dx}\left(e^{x^3}\cos\left(\frac{x}{4} + \frac{1}{2}\right)\right) \]

\[ \text{> value}(\%); \]

\[ 3x^2e^{x^3}\cos\left(\frac{x}{4} + \frac{1}{2}\right) - \frac{1}{4}e^{x^3}\sin\left(\frac{x}{4} + \frac{1}{2}\right) \]

Again, this is an expression for the derivative and not a function. The \texttt{unapply} command can be used to convert it to a function. The third and final way to take a derivative in Maple returns the result as a function. We will, again, take the derivative of

\[ f(x) = x^3\sin(x) \]

First define the function.

\[ \text{> f:=x->x^3*sin(x);} \]

\[ f := x \rightarrow x^3\sin(x) \]

Now execute the following command.
> \texttt{D(f)};

\[
x \to 3 x^2 \sin(x) + x^3 \cos(x)
\]

Note the output. It is taking \( x \) to the expression for the derivative, that is, it is returning a function. If we set the \texttt{D} command equal to a function name, like \texttt{vel}, we now have a derivative function. That is,

\[
> \texttt{vel:=D(f)}; \\
vel := x \to 3 x^2 \sin(x) + x^3 \cos(x)
\]

\[
> \texttt{vel(4)}; \\
48 \sin(4) + 64 \cos(4)
\]

\[
> \texttt{evalf(\%)}; \\
-78.15971151
\]

**More on Plots**

You can plot several functions together by placing your list of functions in \([ \ ]\) in the \texttt{plot} command, for example,

\[
> \texttt{plot([x^3-x^2, \sin(x), \cos(x)],x=-3..3,y=-5..5)};
\]

Hence to graph a function along with its derivative we simply need to define the function

\[
> \texttt{f:=x->x^3-3*x+4}; \\
f := x \rightarrow x^3 - 3 x + 4
\]

and then execute the following command.

\[
> \texttt{plot([f(x),diff(f(x),x)],x=-3..3,y=-10..10)};
\]
More on fsolve

One option that comes in handy with the fsolve command is the ability to restrict the domain of the function. That is, you can look for numeric solutions to equations over an interval that you define. For example, if we just used the fsolve command to find where the \( \sin(x) \) was zero we would get the following.

\[
\text{fsolve}(\sin(x)=0,x);
\]

\[
0.
\]

Since we know that there are an infinite number of solutions to this equation, Maple clearly did not get them all. If we graph the function we can see approximately where the zeros are.

\[
\text{plot}(\sin(x),x=-10..10, y=-3..3);
\]

Since there appears to be a zero between 2 and 4 we could use the command

\[
\text{fsolve}(\sin(x)=0,x=2..4);
\]

\[
3.141592654
\]
to find an approximation to that value. Similarly, we can find the values between 6 and 7, and -4 and -3 by the following two commands.

\[ > \text{fsolve}(\sin(x)=0,x=6..7); \]
\[ > 6.283185307 \]

\[ > \text{fsolve}(\sin(x)=0,x=-4..-3); \]
\[ > -3.141592654 \]

Note that if there is not a zero in the given range then Maple simply tells you that is can not do the calculation. For example,

\[ > \text{fsolve}(\sin(x)=0,x=1..2); \]
\[ > (\text{fsolve},\text{fsolve}(\sin(x)=0,x=1..2),\text{fsolve}(\sin(x)=0,x=1..2)) \]

**Lab Assignment**

1) Exploring the relation between a function and its derivative.
   a) Define the function
   \[ x^3 - 3x^2 + x - 4 \]
   b) Use Maple to find its derivative.
   c) Graph the function and its derivative on the same graph.
      i) On what interval(s) is graph of the derivative is positive? What is happening to the function on this interval?
      ii) On what interval(s) is graph of the derivative is negative? What is happening to the function on this interval?
      iii) Where is the graph of the derivative zero? (find the exact values by using the solve command) What is happening to the function at these points?
      iv) Find the equations to the tangent lines to the function at each point where the graph of the derivative is zero.
      v) Find the equations to the tangent lines to the function at each point where the graph of the derivative is 10.
      vi) Graph the function, its derivative, and the tangent lines you constructed above.

2) Exploring the relation between a function and its derivative.
   a) Define the function
   \[ |\cos(x)| \]
   b) Use Maple to find its derivative.
   c) Graph the function and its derivative on the same graph, on the interval \([-4,4]\).
      i) On what interval(s) is graph of the derivative is positive? What is happening to the function on this interval?
      ii) On what interval(s) is graph of the derivative is negative? What is happening to the function on this interval?
iii) Where is the graph of the derivative zero? (find the exact values by using the solve command) What is happening to the function at these points?

iv) Where is the graph of the derivative not defined? What is happening to the function at these points?

v) Find the equations to the tangent lines to the function at each point where the graph of the derivative is zero.

vi) Find the equations to the tangent lines to the function at each point where the graph of the derivative is \( \frac{1}{2} \).

vii) Graph the function, its derivative, and the tangent lines you constructed above.

3) Experimentation with derivatives.
   a) Define the three functions below as \( f \), \( g \), and \( h \).

   \[
   f(x) = x^3 - 3x^2 + x - 4, \quad g(x) = 2^x, \quad h(x) = \sin(x)
   \]

   b) Find the derivatives of each of the above functions.

   c) Use the diff command to find the derivative of \( f(x)g(x)h(x) \). From the output what do you conjecture is the formula for the derivative of the product of three functions? Write your conjecture in terms of the original functions and their derivatives. In other words, using the symbols \( f(x), f'(x), g(x), g'(x), \) …

   d) Define another function, call it \( j \), as

   \[ \ln(x^3 - x) \]

   e) Use the diff command to find the derivatives of \( f(x)g(x)j(x), f(x)j(x)h(x) \) and \( j(x)g(x)h(x) \). Does your conjecture from (c) still work. If not, make another conjecture to the derivative of a product of three functions.

   f) Define another function, call it \( k \), as

   \[ \sinh(x) \]

   g) Use the diff command to find the derivatives of \( f(x)g(x)h(x)j(x), f(x)g(x)h(x)k(x) \) and \( j(x)g(x)h(x)k(x) \). From the output what do you conjecture is the formula for the derivative of the product of four functions? Write your conjecture in terms of the original functions and their derivatives. In other words, using the symbols \( f(x), f'(x), g(x), g'(x), \) …

   h) Conjecture a general formula for the derivative of \( n \) functions and test it on the product \( f(x)g(x)h(x)j(x)k(x) \).