**Introduction and Goals**
In this lab you will explore one of the coolest commands Maple has to offer the beginner—the plot command. By hand it would take you several minutes to generate the pictures that Maple can produce in seconds. This powerful tool needs to be used wisely to produce accurate results, in this lab we will lay the foundation for using this command.

**Before you start**
Review the basic commands and other Maple features you learned about from the introduction lab. Make sure you review what a function is and understand the process involved to generate a graph by hand.

**Textbook Correspondence**

**Maple Commands and Packages Used**

Packages
Commands: plot,

**History**

**Maple Commands- exploring the plot command**

**Basic structure**
The basic syntax, or way of talking to Maple, for generating a plot of an expression expr is

\[ \text{plot}(\text{expr}, x=a..b, y=c..d, \text{options}); \]

Where options include such things as the color of the lines, the axes labels, the style of the lines, titling the plot, etc..

At its most basic, we can issue the plot command

\[ \text{plot}(\text{expr}, x); \]

and Maple will plot the given expression on its default viewing window, \(-10 \leq x \leq 10\).

**Example**
Suppose we wanted to plot the function \( f(x) = \sqrt{x} \). First, I will get a feel for what the graph looks like with the basic command:

\[ > \text{plot}(\text{sqrt}(x), x); \]
Looking at this graph I see that Maple did not plot any points for negative $x$-values. Is this a mistake? No, if you recall the domain of the square root function is non-negative values. When you tell Maple to plot a function over a given domain, it will pick $x$-values from that domain, evaluate the function at them—producing an ordered pair that it will then plot.

The other thing to notice is that the graph is not “to scale”, meaning that 1 unit on the $x$-axis is not the same as 1 unit on the $y$-axis. We can ask Maple to give us a graph to scale, or at a 1:1 ratio. After you plot the graph, click on it. On your tool bar you will now see a button with 1:1 on it, click it and here you go:

You may be thinking, why does Maple not default to an equal $x$ and $y$ scales. Consider the simple exponential function $y = 2^x$, which grows big really fast.

The plot of $y = 2^x$ where axes are not to scale  

The plot of $y = 2^x$ where axes are 1:1
Choosing your domain values: $x=a..b$

The second component in the plot command is the statement $x=a..b$. This tells Maple your viewing domain, which $x$-values you wish to plot the function for. The default is $x=-10..10$ which will be given to you if you merely type `plot(expr,x)` and leave off the values of $a$ and $b$.

Not all graphs will exhibit their interesting behavior, such as turns and roots, within the default viewing domain $[-10,10]$. Consider $f(x) = x^2 - 194x + 760$:

From algebra, recall that a quadratic equation such as this should be a parabola.

This plot looks like a line, what happened?

Recall from algebra that a polynomial of degree 2 should have 2 roots. This would mean either a real root of multiplicity 2 or 2 real roots or 2 complex roots. I am going to using the `fsolve` command we learned about in the first lab to find out what the roots are:

```
> fsolve(x^2-194*x+760,x);
4.000000000, 190.0000000
```

There is the problem. The graph should cross the $x$-axis at $x=4$ and $x=190$. Since my viewing domain does not include both roots, I do not get an accurate picture of the shape of the graph. Changing my domain:

```
> plot(x^2-194*x+760,x=-10..210);
```
You can also tell Maple to plot arbitrary large values of $x$ by typing in the viewing domain command: $x=a..\text{infinity}$

\begin{verbatim}
> plot((x^2-4)/(2*x^2-10),x=-10..infinity);
\end{verbatim}

Now I can see both roots and the general shape of the graph.

Note that now it is harder to judge the exact place the graph crosses the $x$-axis.

This lets you explore the behavior of the graph for large values of $x$ (the end behavior for positive $x$). You can also explore what happens for really large values of $x$ by typing in $-\text{infinity}$.

\begin{verbatim}
> plot((x^2-4)/(2*x^2-10),x=-infinity..infinity);
\end{verbatim}
Limiting the viewing range
When you choose your viewing domain $x=a..b$, Maple will automatically choose your viewing range unless you use the option $y=c..d$.

> plot(x^3-8*x^2+11*x+20,x);

Suppose I wanted to focus on the roots of this graph. We can change the viewing domain a little on the left but that really will not make much of a difference.

Instead, I will restrict the viewing range to be around the $x$-axis

> plot(x^3-8*x^2+11*x+20,x,y=-10..50);

Now I can see all three root as well as all of the turns in the graph.
Example: The function \( f(x) = \frac{1}{x} + x^2 - x^3 \) is not defined at zero, yet the following plot appears to have plotted something at \( x=0 \).

\[
> \text{plot}(1/x+x^2-x^3, x);
\]

If we restrict the range a bit, say to between -10 and 10 for starters:

\[
> \text{plot}(1/x+x^2-x^3, x, y=-10...10);
\]

We get a better idea of what is happening near \( x=0 \). This view actually reveals that the function is not defined at zero and that for really small positive \( x \)-values the output is very large and for really small negative \( x \)-values the output is negative and very large in magnitude. We would say that the graph has a vertical asymptote at \( x=0 \).

Plotting more than one expression on the same axes
Suppose you wanted to plot more than one graph on the same axes. There are multiple ways to do this, one way being to enclose the functions in square brackets (as a list or set) in the plot command.

\[
> \text{plot}([x^2, x+20], x);
\]
Notice we could have also defined the functions first and then put their names in the plot command. The same graph would result.

\[ f := x \rightarrow x^2; \]
\[ g := x \rightarrow x + 20; \]
\[ \text{plot}([f(x), g(x)], x); \]

To plot more than one expression, you must enter the expressions or a set or list. One way to do this is to enclose the multiple functions in square brackets separated by commas as we did above. Below we will look at different options available to the plot command.

Options: discont

Another important attribute of a graph to consider is x-values that are not in the domain. If we try to plot a function over an interval that contains a value that is not in the domain, Maple will sometimes try to connect the graph on either side of this x-value. For the function \( y = 1/(x-3) \), we know that \( x = 3 \) is not in the domain. Look at what happens:

\[ \text{plot}(1/(x-3), x, y=-15..15); \]
This vertical line at $x=3$ is not part of the graph, it is a vertical asymptote. To get Maple to leave off the vertical asymptote, try:

\[
> \text{plot}(1/(x-3), x, y=-15..15, \text{discont=true});
\]

Notice where we place this plotting option, after the $x$ and $y$ viewing window is defined. This is where all options are placed.

By telling Maple discont=true, you are asking the computer to evaluate all the potential places that the function has a discontinuity and then to plot points on $x$-intervals that leave out these points. So, for our function above, Maple sees a potential discontinuity at $x=3$ and so plots points on the intervals $[-10,3)$ and $(3,10]$. This way, the computer will
not want to connect points on either side of \( x=3 \) because it sees it as 2 different plotting domains.

**Options: style and linestyle**

These are different options and are easy to confuse. *Style* tells Maple how to interpolate between the plotted whereas *linestyle* controls the dash pattern used to render lines in the plot.

At the beginning, the two most important style options are **POINT**, which plots points only, and **LINE** which interpolates between the points and connects the points with a line.

```maple
> plot(0.5^x+100,x,style=POINT);
```

Maple plots the points it has evaluated and does not connect them with a continuous curve.

```
Right now the point style is Maples default, this too can be changed- look in the options menu.
```

The default option for style is “LINE” and is what Maple will give you unless you tell it otherwise. You may never want to use the point option for style.

The linestyle option assumes that you are using the default style option of LINE and controls how the line is rendered on the plot. The four linestyle options are: **SOLID, DOT, DASH, DASHDOT**

Example

```maple
> plot(sqrt(x+9),x,linestyle=DOT);
```
This is especially helpful if you have more than one graph on the plot, in order to
distinguish between the graphs.

\[
> \text{plot}([\sqrt{x+9}, \sqrt{x+6}, \sqrt{x-2}], x, \text{linestyle}=[\text{DASH, DASHDOT, DOT}]);
\]

Maple also has a default color option, the first graph is red, the second green, the third
yellow and so on. You can change this and may want to (yellow does not show up well).
We look at this next.

**Options: color**

Maple has a variety of color choices. Some of the colors are: black, coral, cyan, maroon,
orange, and turquoise.
Options: label and labeldirections

There may be times when you want to label the axes of your plot. The basic form is

\texttt{labels=\{"x axis label", "y axis label"\}}.

Example:

\begin{verbatim}
> plot([sqrt(x+9),sqrt(x+6),sqrt(x-2)],x,linestyle=[DASH,DASHDOT,DOT],
color=[aquamarine,magenta,navy]);
\end{verbatim}

\begin{verbatim}
> plot(0.5^x,x,linestyle=DOT,labels=["this is the x-axis",
"this is the y-axis"]);
\end{verbatim}

Note: the labels must be enclosed in "quotes"

We can also change the direction that the labels are printed in- this is done with the labeldirections option. There are only 2 ways to print them, horizontally (which is the default as above) or vertically.
Remember these are all optional, if you do not use them Maple will use the default options. Sometimes, that is sufficient and there is no need to mess with the other options.

Options: legend
This tells Maple to provide a legend, like a map legend, that lets the viewer know which graph is which. It is especially useful when you have multiple plots.

> plot([sqrt(x+9), sqrt(x+6)], x, linestyle=[DASH, DOT], legend=["sqrt(x+9)", "sqrt(x+6)"]);
Options: title
This is a nice feature and lets you label the graph of your function. When writing lab reports and turning in multiple graphs it is often necessary to label the graph so that everyone knows what it is a picture of.

\[ \texttt{plot(0.5^x,x,linestyle=DOT,title="the plot of } y=0.5^x, \text{ the decay function")}; \]

Maple prints out the legend below the graph, labeling the first function with the red dashed line and the second function with the green dotted line.

Pre-defined functions and constants
Maple has many pre-defined functions, below is a list of some.

<table>
<thead>
<tr>
<th>Function or constant</th>
<th>How to speak to Maple</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{x})</td>
<td>sqrt(x)</td>
</tr>
<tr>
<td>(</td>
<td>x</td>
</tr>
<tr>
<td>(\pi)</td>
<td>Pi</td>
</tr>
<tr>
<td>(e^x)</td>
<td>exp(x)</td>
</tr>
</tbody>
</table>

**Trigonometric functions**

| cos(x), sin(x), tan(x), cot(x), sec(x), csc(x) |

**Exercises**

Part 1- using the plot command

1. Issue the following commands into Maple and answer the questions that follow.

```maple
> f:=x->sqrt(x^2*sin(x+Pi/6));
```

**question 1a** Does it look like this graph has any x-intercepts?

```maple
> fsolve(f(x),x);
```

**question 1b** What does the solve command say about the zeroes of the function?

```maple
plot(f(x),x=-4..4);
```

**question 1c** Does it look like there is an x-intercept in this interval?

```maple
plot(f(x),x=-6..-3);
```

**question 1d** Does it look like there is an x-intercept in this interval?

**question 1e** Go back and look at the graph, why do you think Maple does not show these zeroes?

```maple
g:=x->x^2*sin(x+Pi/6);
plot(g(x),x);
```

**question 1f** Why should \(f(x)\) have the same zeroes as \(g(x)\)?

```maple
> fsolve(f(x),x=-4..-3);
> fsolve(f(x),x=2..4);
> fsolve(f(x),x=5..7);
```

**question 1g** When \(g(x)\) is negative, what is true about \(f(x)\)?

```maple
plot(f(x),x=-10..10,numpoints=900);
plot(f(x),x=-20..20,numpoints=900);
plot(f(x),x=-20..20,numpoints=1500);
plot(f(x),x=-50..50,numpoints=5500);
```

**question 1h** What do the prior 3 commands tell you about \(f(x)\) and some of its x-intercepts?

The `numpoints` option in the `plot` command instructs Maple to use more points when deciding how to plot the function. There is a balance between just enough and the time it will take the computer to evaluate all those points.

**question 1i** Looking at the first plot, what are the approximate zeroes of \(f(x)\)?

**question 1j** Is it hard to see the true behavior of the function in the second plot?
**question 1k** Considering plots 1, 3, and 4 what do you think the overall behavior of the function is? How many roots does it have?

2. **For the following functions, use Maple’s plot command to produce a graph that shows all relevant behavior of the function such as turns, roots, high and low points, asymptotic behavior.** This may mean that you have to adjust the viewing domain and/or range. Remember to use the *fsolve* command to help find all roots as well as look at what different choices of the viewing domain do to the appearance of the graph.

Your graph should include a title that contains the function rule.

a) \( f(x) = x^2 + 1/x + 1/x^2 \)
b) \( g(x) = 0.25x^4 - 0.913x^3 + x^2 - 0.175x - 0.5 \)

3. **Plot the following sets of functions on the same axes.** Provide both a legend and title for your plot and use the colors blue, magenta, navy and green. Use different line styles for each function. Also make your viewing domain [-30,30] and appropriately limit the range to see the graphs. If appropriate, use the *discont=true* option.

   a) \( y = x, y = -x, y = 0.5x, y = -3x \)
   b) \( y = x^2, y = x^3 \)
   c) \( f(x) = \ln(x), f(x) = e^x \)
   d). \( f(x) = \frac{1}{x}, f(x) = \frac{1}{x^2} \)
   e). \( f(x) = x^{\frac{1}{3}}, f(x) = x^{\frac{2}{3}} \)

**Part 2- Shifts, translations**

*Linear functions* \( y = ax + b \)

From algebra you might recall the effect that the values of \( a \) and \( b \) have on the line. If not, plot several example lines with Maple until you can answer the following.

1. **When the value of \( a \) is negative, what happens to the line? When the value of \( a \) is positive what happens to the line? When the value of \( a \) is zero, what happens to the line?**

2. **What effect does the value of \( b \) have on the line?**

3. **What is the general equation of a horizontal line? What is the general equation of a vertical line?**
**Quadratic functions** \( y=ax^2+bx+c \)

When working with parameters and their effect on a graph or behavior, it is always good to consider negative, zero, fractional, and positive values. Depending on the context, other values should be considered as well.

Recall that a quadratic can be put into the form \( y = a(x-h)^2 + k \) by a process called completing the square.

Maple also has a nice feature in its student package called `completesquare` that will complete the square on a general quadratic.

\[ > \text{completesquare}(a*x^2+b*x+c,x); \]

\[ a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \]

1. When we complete square on the general quadratic \( y = ax^2 + bx + c \), we get the above equation in the form: \( y = a(x-h)^2 + k \). In terms of the coefficients \( a,b,c \), what is \( h \)? What is \( k \)? What is \( a \)?

2. Plot the following pairs of quadratics on the same axis and then answer the questions that follow.
   a. \( f(x) = x^2, \ g(x) = 3x^2 \)
   b. \( f(x) = x^2, \ g(x) = (x-3)^2 \)
   c. \( f(x) = x^2, \ g(x) = x^2 + 3 \)
   d. \( f(x) = x^2, \ g(x) = (x+3)^2 \)

1. What effect does "a" have on the graph? Include graphs in your response.
2. What are "h" and "k"? Include graphs in your response.

*For question 3-7, by related we mean how are their roots, their high points, their low points related?*

3. How are the graphs of \( y = x^2 \) and \( y = 3x^2 \) related?
4. How are the graphs of \( y = x^2 \) and \( y = (x-3)^2 \) related?
5. How are the graphs of \( y = x^2 \) and \( y = x^2 + 3 \) related?
6. How are the graphs of \( y = x^2 \) and \( y = (x+3)^2 \) related?
7. Use your answers to questions 3-6 to explain how the graphs of \( y = x^2 \) and \( y = a(x-h)^2 + k \) are related.

**General function** \( f(x) \)

Answer the following: Use the functions \( y = x^2, y = x^3 \) and \( y = \sin(x) \) to help you.

Given the graph of \( f(x) \) and \( k>1 \), how do you find the relevant aspects (such as roots, high and low points) of the graph of

a) \( f(x+k) \)
b) \( f(x-k) \)
c) \( f(kx) \)
d) \( f(x)+k \)
e) \( f(x)-k \)
f) \( kf(x) \)
g) \( (1/k)f(x) \)
h) \( f((1/k)x) \)