1. Find all of the critical values, intervals where the function is increasing and decreasing, all relative extrema, intervals where the function is concave up and concave down and all inflection points of the following function.

\[ f(x) = x^4 + 4x^3 + 1 \]

**Solution:** For the first part we will need the first derivative.

\[ f'(x) = 4x^3 + 12x^2 \]

Solving this for 0 gives the critical values of \( x = 0 \) and \( x = -3 \). Plotting these on the number line and determining the sign of the derivative in each subinterval gives

\[
\begin{array}{c|c|c}
- & + & + \\
-3 & 0 & \\
\end{array}
\]

To do the second part we need the second derivative.

\[ f''(x) = 12x^2 + 24x \]

Solving this for 0 gives the values of \( x = 0 \) and \( x = -2 \). Plotting these on the number line and determining the sign of the second derivative in each subinterval gives

\[
\begin{array}{c|c|c}
+ & - & + \\
-2 & 0 & \\
\end{array}
\]

So in conclusion we have critical values at \( x = 0 \) and \( x = -3 \). The function is increasing on \((-3, \infty)\) and decreasing on \((-\infty, -3)\). There is a relative minimum at \( x = -3 \) and no relative maximums. The function is concave up on \((-\infty, -2) \cup (0, \infty)\) and concave down on \((-2, 0)\). There are points of inflection at \( x = 0 \) and \( x = -2 \).