1. (15 points) Find $f'(x)$, do not simplify your answer

$$f(x) = \left(e^x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right)$$

Solution:

$$f'(x) = \left(e^x + \frac{1}{x}\right)\left(-\frac{2}{x^3}\right) + \left(e^x - \frac{1}{x^2}\right)\left(1 + \frac{1}{x^2}\right)$$

2. (15 points) Find $f'(x)$, do not simplify your answer

$$f(x) = \frac{-4\ln(x)}{x^4 + 3}$$

Solution:

$$f'(x) = \frac{-\frac{4}{x}(x^4 + 3) + 16x^3 \ln(x)}{(x^4 + 3)^2}$$

3. (15 points) Find $f'(x)$, do not simplify your answer

$$f(x) = (7x + 3)^3(x^2 - 4)^6$$

Solution:

$$f'(x) = 12x(7x + 3)^3(x^2 - 4)^5 + 21(7x + 3)^2(x^2 - 4)^6$$

4. (15 points) Find $f'(x)$, do not simplify your answer

$$f(x) = 7x^7$$

Solution:

$$f'(x) = 7x^67x^7\ln(7)$$

5. (15 points) Given the demand curve $x = 26 - 7p$, find the elasticity. If the price were set at $p = 2$, would the demand be elastic or inelastic? Find the price that will maximize revenue.

Solution:

$$E = \frac{p}{x} \frac{dx}{dp} = \frac{7p}{26 - 7p}$$

So $E(2) = \frac{14}{12} = \frac{7}{6}$ and hence the demand is elastic. The price that will maximize revenue is when $E = 1$ and hence $p = \frac{26}{14} = \frac{13}{7} = 1.857142857$. 

6. (20 points) Consider the function \( f(x) = 3x^5 - 40x^3 + 20 \), answer the following questions and in each case give some supporting work to justify your answer. Use algebra and calculus to answer these questions, estimating the answers from the function’s graph will not be accepted.

**Solution:**

(a) The domain of the function is \( \mathbb{R} \).
(b) Is the function is not symmetric with respect to either the y-axis or the origin.
(c) There are no vertical asymptotes.
(d) There are no horizontal asymptotes.
(e) The y-intercept is \( f(0) = 20 \).
(f) The critical values are \( 0, 2\sqrt{2} \) and \( -2\sqrt{2} \).
(g) The function is increasing on \( (-\infty, -2\sqrt{2}) \) and \( (2\sqrt{2}, \infty) \). The function is decreasing on \( (-2\sqrt{2}, 2\sqrt{2}) \).
(h) There is a relative maximum at \( x = -2\sqrt{2} \) and a relative minimum at \( x = 2\sqrt{2} \).
(i) The function is concave up on \( (-2, 0) \) and \( (2, \infty) \). The function is concave down on \( (-\infty, -2) \) and \( (0, 2) \).
(j) The inflection points are at \( x = -2, x = 0 \) and \( x = 2 \).

7. (15 points) What is the area of the largest rectangle that can be enclosed by a circle of radius \( a \)?

**Solution:** The rectangle is a square with sides of length \( \sqrt{2} a \) and hence area \( 2a^2 \).