Quiz #3 Key

1. Using the limit formula derived in class, find the instantaneous rate of change of \( f(x) = x^2 - 4 \) with respect to \( x \) at \( x = c = 2 \).

Solution:

\[
\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{(2+h)^2 - 4 - 0}{h} = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} = \lim_{h \to 0} \frac{4h + h^2}{h} = \lim_{h \to 0} 4 + h = 4
\]

2. Using the limit formula derived in class, find the instantaneous rate of change of \( f(x) = x^3 \) with respect to \( x \) at \( x = c = 3 \).

Solution:

\[
\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{(3+h)^3 - 27}{h} = \lim_{h \to 0} \frac{27 + 27h + 9h^2 + h^3 - 27}{h} = \lim_{h \to 0} \frac{27h + 9h^2 + h^3}{h} = \lim_{h \to 0} \frac{h(27 + 9h + h^2)}{h} = \lim_{h \to 0} 27 + 9h + h^2 = 27
\]