1. A fence is to be built around a 200-square-foot rectangular field. Three sides are to be made of wood costing $10 per foot, while the other side is made of stone costing $30 per foot. Find the dimensions of the enclosure that minimizes total cost.

**Solution:** Let the sides of the rectangle be labeled $x$ and $y$ then the cost of the fence is $C = 10x + 30x + 10y + 10y = 40x + 20y$. Since the area must enclose 200 square feet we also have that $xy = 200$. Solving the second equation for $y$ gives $y = \frac{200}{x}$ and hence our cost function is $C(x) = 40x + \frac{4000}{x}$. The interval of concern for $x$ in this problem is $(0, \infty)$. Since the function approaches infinity as $x$ approaches either 0 or infinity we know that the minimum will occur at a critical point. $C'(x) = 40 - \frac{4000}{x^2}$ which is defined everywhere on $(0, \infty)$. The derivative is zero when $x = 10$ and the first derivative test confirms that this is indeed a minimum. So the dimensions that minimize the cost are $x = 10$ and $y = 20$.

2. Find $$\int 4x^2 + \frac{3}{x} \, dx$$

**Solution:**

$$\int 4x^2 + \frac{3}{x} \, dx = \int 4x^2 \, dx + \int \frac{3}{x} \, dx$$

$$= 4 \int x^2 \, dx + 3 \int \frac{1}{x} \, dx$$

$$= \frac{4x^3}{3} + 3\ln|x| + C$$

$$= \frac{4}{3}x^3 + 3\ln|x| + C$$