1. Find \( \frac{dy}{dx} \) given that \( y = x^{x^x} \)

**Solution:** First note that if \( y = x^{g(x)} \)

\[
\ln (y) = \ln \left( x^{g(x)} \right) \\
= g(x) \ln (x)
\]

So

\[
\frac{y'}{y} = g(x) \frac{1}{x} + g'(x) \ln (x) \\
\]

\[
y' = x^{g(x)} \left( g(x) \frac{1}{x} + g'(x) \ln (x) \right)
\]

Now

\[
\frac{d}{dx} \left( x^{x^x} \right) = x^{x^x} \left( x^x \frac{1}{x} + \frac{d}{dx} \left( x^{x^x} \right) \ln (x) \right)
\]

\[
= x^{x^x} \left( x^x \frac{1}{x} + \left( x^{x^x} \left( x^x \frac{1}{x} + \frac{d}{dx} (x^x) \ln (x) \right) \right) \ln (x) \right)
\]

\[
= x^{x^x} \left( x^x \frac{1}{x} + \left( x^{x^x} \left( x^x \frac{1}{x} + x^x \left( x^{1+\ln (x)} \right) \ln (x) \right) \right) \ln (x) \right)
\]

\[
= x^{x^x} \left( x^x \frac{1}{x} + \left( x^{x^x} \left( x^x \frac{1}{x} + x^x \left( 1 + \ln (x) \right) \ln (x) \right) \right) \ln (x) \right)
\]

\[
= x^{x^x} \left( x^x \frac{1}{x} + \left( x^{x^x} \left( x^x \frac{1}{x} + x^x \ln (x) + x^x \ln^2 (x) \right) \right) \ln (x) \right)
\]

\[
= x^{x^x} x^{x-1} + x^{x^x} x^{x^x} x^{x-1} \ln (x) + x^{x^x} x^{x^x} x^x \ln^2 (x) + x^{x^x} x^{x^x} x^x \ln^3 (x)
\]

\[
= x^{x^x} + x^{x-1} + x^{x^x} x^x + x^{x^x} x^x \ln (x) + x^{x^x} x^x \ln^2 (x) + x^{x^x} x^x \ln^3 (x)
\]