Name:

Write all of your responses on the exam paper. If you need more room please use the backs of the exam pages. Make sure that you show all of your work. Answers without supporting work will receive no credit. Remember that there is no sharing of calculation devices on this exam.

1. (10 points) Find the following limit

\[ \lim_{h \to 0} \frac{(1 + h)^4 - 1}{h} \]

2. (10 points) Find the following limit

\[ \lim_{x \to 0} \cos \left( \frac{x^2 - 1}{x^2 + 1} \right) \]
3. (15 points) Find the following limit

\[ \lim_{x \to \infty} \left( \sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right) \]

4. (15 points) Using the definition of the derivative find \( f'(x) \) of

\[ f(x) = x^2 - 3x + 7 \]
5. (10 points) Find $f'(x)$ of

$$ f(x) = \frac{\sin(x) + 3}{x^2 + x + 1} $$

6. (10 points) Find $f'(x)$ of

$$ f(x) = \sin(x) \cos(x)(x^2 + 3) $$
7. (15 points) Find $f'(x)$ of 

$$f(x) = (3x^4 + 7x - \ln(x^3 - 2x + 1))^7(3^x - \cos(sin(x)))$$

8. (15 points) Find $y'$ of 

$$xy^2 - 2y^3 + \frac{x}{y} = \sin(y)$$
9. (15 points) Find the equation to the tangent line to the curve \( y = x^2 \sqrt{x - 4} \) at the point (5, 25).

10. (15 points) Find the following for \( f(x) = 2x^3 - 7x^2 + 4x + 5 \),
    
    (a) Intervals where the function is increasing.
    
    (b) Intervals where the function is decreasing.
    
    (c) Intervals where the function is concave up.
    
    (d) Intervals where the function is concave down.
    
    (e) Local maximums.
    
    (f) Local minimums.
    
    (g) Points of inflection.
11. (15 points) The upper left-hand corner of a piece of paper 8\(\frac{1}{2}\) inches wide by 12 inches long is folded over to the right-hand edge as in the picture below. How would you fold it so as to minimize the length of the fold? That is, how would you choose \(x\) to minimize \(y\). Hints: First, remember that minimizing \(y^2\) is equivalent to minimizing \(y\). That is, if you find the \(x\) value that minimizes \(y^2\) it will also minimize \(y\). Second, once you find an equation for \(y^2\) in terms of \(x\) simplify it completely before minimizing it.
12. (15 points) A street light is mounted at the top of a 20 foot pole. A man 5 feet 11 inches tall walks away from the pole at a speed of 6 feet per second. How fast is the tip of his shadow moving when he is 34 feet from the pole?

13. (15 points) Use Newton’s method to approximate all of the solutions to the following equation to eight decimal places.

\[ \sqrt{x^2 - x + 1} = 2\sin(\pi x) \]
14. (10 points) Find
\[ \int 3x^4 - 2x^3 + \sin(x) - \frac{4}{x} + \frac{7}{x^2} \, dx \]

15. (15 points) Find
\[ \int \sqrt{x^3 + 1} \, x^5 \, dx \]