1. (10 points) Consider the following function

\[ f(x) = \ln(x^2 - 3x + 1) \]

(a) Graph the following function on \([-10, 10] \times [-5, 5]\) and draw the graph on the axes below.

(b) Just looking at the graph, what is the domain of this function? The domain is approximately \((-\infty, 0.5) \cup (2, \infty)\)

(c) Graph both \(f(x)\) and the function \(x^2 - 3x + 1\) on the same set of axes and draw the graph on the axes below.

(d) What are the exact solutions to the equation \(x^2 - 3x + 1 = 0\). Also, for what values of \(x\) is the expression \(x^2 - 3x + 1 > 0\) and for what values of \(x\) is the expression \(x^2 - 3x + 1 < 0\)?

The exact solutions to \(x^2 - 3x + 1 = 0\) are \(x = \frac{3}{2} + \frac{\sqrt{5}}{2}\) and \(x = \frac{3}{2} - \frac{\sqrt{5}}{2}\). The inequality \(x^2 - 3x + 1 > 0\) is satisfied on the intervals \((-\infty, \frac{3}{2} - \frac{\sqrt{5}}{2})\) and \((\frac{3}{2} + \frac{\sqrt{5}}{2}, \infty)\). Also, the inequality \(x^2 - 3x + 1 < 0\) is satisfied on the interval \((\frac{3}{2} - \frac{\sqrt{5}}{2}, \frac{3}{2} + \frac{\sqrt{5}}{2})\).

(e) What is the domain of \(\ln(x)\)? The domain is \((0, \infty)\)
(f) Algebraically, what is the exact domain of \( f(x) \)? The domain is \((-\infty, \frac{3}{2} - \frac{\sqrt{5}}{2}) \) and \( \left( \frac{3}{2} + \frac{\sqrt{5}}{2}, \infty \right) \).

(g) Does the function \( y = \ln(x) \) have any vertical asymptotes? If so, where? Yes, at \( x = 0 \).

(h) Does the function \( f(x) \) have any vertical asymptotes? If so, where? Yes, at \( x = \frac{3}{2} - \frac{\sqrt{5}}{2} \) and \( x = \frac{3}{2} + \frac{\sqrt{5}}{2} \).

2. (10 points) Consider the following function

\[
g(x) = \frac{\sqrt{1+x} - 3}{\sqrt{x-4} - 2}
\]

(a) Fill in the following charts

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 + \frac{1}{10}</td>
<td>0.6689545500</td>
<td>8 + \frac{1}{100}</td>
<td>0.6643241270</td>
</tr>
<tr>
<td>8 + \frac{1}{1000}</td>
<td>0.6666906682</td>
<td>8 - \frac{1}{10}</td>
<td>0.6666435178</td>
</tr>
<tr>
<td>8 + \frac{1}{10000}</td>
<td>0.6666800000</td>
<td>8 - \frac{1}{100}</td>
<td>0.6666643518</td>
</tr>
<tr>
<td>8 + \frac{1}{100000}</td>
<td>0.6666800000</td>
<td>8 - \frac{1}{1000}</td>
<td>0.6666643518</td>
</tr>
<tr>
<td>8 + \frac{1}{1000000}</td>
<td>0.6666800000</td>
<td>8 - \frac{1}{10000}</td>
<td>0.6666643518</td>
</tr>
</tbody>
</table>

(b) From the charts above what is the value of

\[
\lim_{x \to 8} \frac{\sqrt{1+x} - 3}{\sqrt{x-4} - 2} \approx 0.66666666\ldots
\]

(c) What is the Maple command to calculate

\[
\lim_{x \to 8} \frac{\sqrt{1+x} - 3}{\sqrt{x-4} - 2}
\]

\[
\text{limit((sqrt(1+x)-3)/(sqrt(x-4)-2),x=8)};
\]

(d) What is the exact value of the following limit (just use the Maple command output)

\[
\lim_{x \to 8} \frac{\sqrt{1+x} - 3}{\sqrt{x-4} - 2} = \frac{2}{3}
\]

3. (10 points) Consider the following function

\[
g(x) = \ln \left( \left| \frac{x-1}{x} \right| \right)
\]

(a) Define this function as \( g \) in Maple and write down the command you used.

\( g := x \rightarrow \ln(\text{abs}(x-1)/x) ; \)

(b) Input the following expression into Maple

\[
g(1/2 + h) - g(1/2)
\]

write down the output Maple gave you.

\[
\ln \left( \left| \frac{-1/2+h}{1/2+h} \right| \right)
\]

(c) Simplify the above expression and write down the output Maple gave you.

\[
\ln \left( \left| x \right| \right)
\]

(d) Take the limit of the above expression as \( h \) approaches 0. What is the result? \(-4\)

(e) What is the slope of the tangent line to the curve \( \ln \left( \left| \frac{x-1}{x} \right| \right) \) at \( x = \frac{1}{2} \)? \(-4\)