1. (10 Points Each) Find the following limits using algebraic techniques, you may not use Bernoulli’s rule (a.k.a. l’Hospital’s rule).

(a) \( \lim_{x \to 5} \frac{\sqrt{4x^2 - 8x + 4}}{x^2 + 3x} \)

(b) \( \lim_{x \to \infty} \sqrt{4x^2 + 7x - 5} - 2x \)
(c) \( \lim_{x \to 2} \frac{\frac{1}{2} - \frac{1}{x}}{2 - x} \)

(d) \( \lim_{x \to \infty} \frac{x \sin(x) + 3}{x - 4} \)
2. (10 Points Each) Find the following limits, you may use Bernoulli’s rule (a.k.a. l’Hospital’s rule) if it applies.

(a) \( \lim_{x \to 0} \frac{\sin(x) - x}{x^3} \)

(b) \( \lim_{x \to \infty} \left( \frac{2x}{2x - 3} \right)^x \)
3. (10 Points) State the definition of the derivative of a function $f(x)$.

4. (15 Points) Find the derivative of the following function using the definition of the derivative.

$$f(x) = \sqrt{2x^2 + 5}$$
5. (5 Points Each) Find the derivatives of the following functions using the derivative rules, do not simplify your answer.

(a) \( f(x) = x^5 + \frac{1}{x} - \frac{1}{x^2} + e^x + \ln(x) - \sin(x) + \sec(x) \)

(b) \( f(x) = x^2 \sin(x) \)
(c) \( f(x) = \frac{x^3 - 2x + 3}{x^4 - 3x^2 + 8} \)

(d) \( f(x) = \cos(x^3 - 2) \)
(e) \( f(x) = x^{\tan(x)} \)

(f) \( f(x) = \frac{x^2 \sin(x) \cos(x)}{(x^3 - 1)(e^{3x} - \ln(x))} \)
6. (15 Points) Find \( \frac{dy}{dx} \) of \( \sin(xy^2) - x^3 y^3 = x^2 + x + 1 \).

7. (10 Points) Find the equation of the tangent line to the curve \( f(x) = \frac{x}{x-3} \) at \( x = 2 \).
8. (15 Points) For the function

\[ f(x) = x^2 e^x \]

Find the domain, intercepts, asymptotes, intervals of increase or decrease, local maximums and minimum values, intervals of concavity and inflection points.
9. (15 Points) A length of pipe is being taken around a 90° corner from an 11 foot hallway to a 7 foot hallway. Assuming the pipe has no width what is the longest pipe that can be taken around this corner? For extra credit assume that the pipe has an outside diameter of 3 inches.
10. (15 Points Each) Find the following.

(a) \[ \int (3x^4 - x^2 + 1 - \frac{1}{x} + \sqrt{x^2}) \, dx \]

(b) \[ \int \frac{4x}{\sqrt{3x^2 + 5}} \, dx \]
11. (10 Points) Do only one of the following.

(a) Given that \( \sinh(x) = \frac{e^x - e^{-x}}{2} \) derive the formulas for \( \sinh^{-1}(x) \) and \( \frac{d}{dx} (\sinh^{-1}(x)) \).

(b) Derive the formula for \( \frac{d}{dx} (\tan^{-1}(x)) \).

(c) Prove \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \).

(d) Using \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \), \( \lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0 \) and the definition of the derivative prove that \( \frac{d}{dx} (\sin(x)) = \cos(x) \).

(e) State and prove the product rule.

(f) State and prove the quotient rule.