1. Find all relative (local) maximums and minimums, intervals of increasing and decreasing, intervals of concave up and concave down and inflection points of the following function. All values must be found using the methods discussed in class and must be exact.

\[ f(x) = x^4 - 4x^3 - 8x^2 - 3 \]

**Solution:** \( f'(x) = 4x^3 - 12x^2 - 16x = 4x(x-4)(x+1) \), so the critical numbers are \( x = 0, 4, -1 \). Using the number line as was done in class we see that the function is increasing on the intervals \((-1, 0)\) and \((4, \infty)\), decreasing on \((-\infty, -1)\) and \((0, 4)\). The function has a local minimum at \( x = -1 \) and \( x = 4 \) and a local maximum at \( x = 0 \).

\( f''(x) = 12x^2 - 24x - 16 \), which has roots at \( 1 + \sqrt{21}/3 \) and \( 1 - \sqrt{21}/3 \). Using the number line as was done in class we see that the function is concave up on the intervals \((-\infty, 1 - \sqrt{21}/3)\) and \((1 + \sqrt{21}/3, \infty)\) and concave down on the interval \((1 - \sqrt{21}/3, 1 + \sqrt{21}/3)\). The function has inflection points at \( x = 1 - \sqrt{21}/3 \) and \( x = 1 + \sqrt{21}/3 \).