1. (10 Points) The following exercises all deal with the curve, $2(x^2 + y^2)^2 = 25(x^2 - y^2)$.

(a) Find all the points on the curve where $x = 1$. Keep your answers in exact form.

Solution: \((1, -\sqrt{-\frac{29}{4} + \frac{5\sqrt{41}}{4}})\) and \((1, \sqrt{-\frac{29}{4} + \frac{5\sqrt{41}}{4}})\).

(b) Find the equation of the tangent line to the curve at each point where $x = 1$. Use approximations to the numbers to at least 7 decimal places.

Solution: 
\[
-0.8682772004 - 0.6469565463(-1 + x) = -0.2213206540 - 0.6469565463x
\]
and 
\[
0.8682772004 + 0.6469565463(-1 + x) = 0.2213206540 + 0.6469565463x
\]

2. (15 Points) The following exercises all deal with the curve, $f(x) = \frac{x}{x^3 + 1}$.

(a) Plot the function in the window $-5 \leq x \leq 5$ and $-1 \leq y \leq 1$. Draw the graph below.

Solution:

(b) From the graph how many tangent lines to the graph pass through the point \((0, 0.5)\)?

Solution: 4

(c) Use Mathematica to find the $x$ values of the point of tangency for each tangent line that passes through the point \((0, 0.5)\). Use approximations to the numbers to at least 7 decimal places.

Solution: $x = -2.597682770850205$, $x = 2.2523752774042145$, $x = 0.7721218374259091$ and $x = -0.5750073720469436$. The commands used here were $f'[x_] := x/(x^3 + 1)$ and $\text{NSolve}[f'[a] (0 - a) + f[a] == 1/2, a]$. 