1. Find all the critical values of the function

\[ f(x) = x \ln(x) \]

**Solution:**

\[ f'(x) = \frac{1}{x} + \ln(x) \]
\[ = \frac{1}{x} + \ln(x) \]

Check where \( f'(x) = 0 \).

\[ 1 + \ln(x) = 0 \]
\[ \ln(x) = -1 \]
\[ x = e^{-1} \]
\[ x \approx 0.3678794411714423216 \]

Check where \( f'(x) \) does not exist. As is easily seen, the derivative does not exist whenever \( \ln(x) \) does not exist, but in this case the function will not exist either. So the only critical value for \( f(x) \) is \( x = e^{-1} \).

2. Find the absolute maximum and minimum of the following function on the interval \([1, 4]\)

\[ f(x) = x - 3 \ln(x) \]

**Solution:** First, find all of the critical values.

\[ f'(x) = 1 - \frac{3}{x} \]

so

\[ 1 - \frac{3}{x} = 0 \]
\[ 1 = \frac{3}{x} \]
\[ x = 3 \]

Also, \( f'(x) \) does not exist when \( x = 0 \). This is not a critical value since the function is not defined there. Now evaluate the function at the critical values and the endpoints.

\[ f(x) = 1 - 3 \ln(1) = 1 \]
\[ f(x) = 3 - 3 \ln(3) \approx -0.2958368660043290742 \]
\[ f(x) = 4 - 3 \ln(4) \approx -0.1588830833596718565 \]

So the absolute maximum is 1 at \( x = 1 \), and the absolute minimum is \( 3 - 3 \ln(3) \) at \( x = 3 \).