1. (10 points) Find \( f'(x) \), do not simplify your answer.

\[
f(x) = x \sin(x)
\]

Solution:

\[
f'(x) = x \sin(x) \left( \cos(x) \ln(x) + \frac{\sin(x)}{x} \right)
\]

2. (10 points) Use differentials (or, equivalently, a linear approximation) to estimate \((8.06)^{2/3}\).

Solution:

\[
(8.06)^{2/3} \approx 8^{2/3} + \frac{2}{3} \cdot 8^{-1/3} (0.06) = 4 + \frac{0.06}{3} = 4.02
\]

3. (10 points) Find the absolute maximum and absolute minimum of \( f(x) = (x^2 - 1)^3 \) on the interval \([-1, 2]\).

Solution: \( f'(x) = 6x(x^2 - 1)^2 \), so the critical values are \( x = 0, x = -1 \) and \( x = 1 \). So \( f(-1) = 0, f(0) = -1, f(1) = 0 \) and \( f(2) = 27 \). Hence the absolute maximum is 27 at \( x = 2 \) and the absolute minimum is -1 at \( x = 0 \).

4. (10 points) Find all of the numbers \( c \) that satisfy the Mean Value Theorem for \( f(x) = x^3 - x^2 + x - 1 \) on the interval \([-2, 3]\).

Solution: \( f'(x) = 3x^2 - 2x + 1 \) and \( \frac{f(3) - f(-2)}{3 - (-2)} = 7 \). The solutions to \( 3x^2 - 2x + 1 = 7 \) are \( x = \frac{1 + \sqrt{19}}{3} \) and \( x = \frac{1 - \sqrt{19}}{3} \).

5. (20 points) Consider the function \( f(x) = 3x^5 - 40x^3 \), answer the following questions and in each case give some supporting work to justify your answer.

(a) The domain of the function.

Solution: \( \mathbb{R} \).

(b) Is the function symmetric with respect to the y-axis or the origin?

Solution: Since \( f(-x) = -f(x) \), the function is symmetric with respect to the origin.

(c) Are there any vertical asymptotes?

Solution: No, it is a polynomial.

(d) Are there any horizontal asymptotes?

Solution: No, it is a polynomial.

(e) What is the y-intercept?

Solution: \( f(0) = 0 \)

(f) What are the x-intercepts?

Solution: \( 3x^5 - 40x^3 = 0 \) when \( x = 0, x = \sqrt[4]{\frac{40}{3}} \) and \( x = -\sqrt[4]{\frac{40}{3}} \).

(g) What are the critical values?

Solution: \( f'(x) = 15x^4 - 120x^2 \), so the critical values are \( x = 0, x = 2\sqrt{2} \) and \( x = -2\sqrt{2} \).

(h) Find the intervals where the function is increasing and decreasing.

Solution: \( f(x) \) is increasing on \((-\infty, -2\sqrt{2}) \) and \((2\sqrt{2}, \infty) \) and decreasing on \((-2\sqrt{2}, 2\sqrt{2}) \).

(i) Find all of the relative extrema.

Solution: \( f(x) \) has a local maximum at \( x = -2\sqrt{2} \) and a local minimum at \( x = 2\sqrt{2} \).

(j) Find the intervals where the function is concave up and concave down.

Solution: \( f(x) \) is concave down on \((-\infty, -2) \) and \((0, 2) \) and concave up on \((-2, 0) \) and \((2, \infty) \).
(k) Find the inflection points.

**Solution:** \( f(x) \) has inflection points at \( x = 0, x = 2 \) and \( x = -2 \).

(l) Draw a rough sketch of the function.

6. **(10 points)** Find the slant asymptote to the function

\[
f(x) = \frac{2x^3 + x^2 + 1}{x^2 + 1}
\]

**Solution:** \( y = 2x + 1 \).

7. **(10 points)** Use both the first and second derivative tests to determine the local maximums and minimums of

\[ f(x) = x + \sqrt{1-x} \]

**Solution:** Note that \( f'(x) = 1 - \frac{1}{2\sqrt{1-x}} \) and \( f''(x) = -\frac{1}{4\sqrt{(1-x)^3}} \). The only critical value is \( x = \frac{3}{4} \) and there is a local maximum at this point.

8. **(10 points)** Which is larger \( e^\pi \) or \( \pi^e \)? Justify your answer.

**Solution:** Recall from class that the function \( f(x) = \frac{\ln(x)}{x} \) has an absolute maximum at \( e \). To see this, note that \( f'(x) = \frac{1-\ln(x)}{x^2} \) which gives us only one critical value, \( x = e \). Since \( f''(x) = \frac{-3+2\ln(x)}{x^3} = -\frac{1}{e^3} < 0 \) we know that it is a maximum. So,

\[
\begin{align*}
\frac{\ln(e)}{e} &> \frac{\ln(\pi)}{\pi} \\
\pi \ln(e) &> e \ln(\pi) \\
\pi \ln(e) &> e \ln(\pi) \\
\ln(e^\pi) &> \ln(\pi^e) \\
e^\pi &> \pi^e
\end{align*}
\]