1. (10 points) Use the definition of the derivative to find \( f'(x) \) for
\[
f(x) = x^3 - 2x^2 + x - 5
\]

Solution:
\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{(x + h)^3 - 2(x + h)^2 + (x + h) - 5 - (x^3 - 2x^2 + x - 5)}{h}
\]
\[
= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x^2 - 4xh - 2h^2 + x + h - 5 - x^3 - 2x^2 - x + 5}{h}
\]
\[
= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 4xh - 2h^2 + h}{h}
\]
\[
= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 - 4x - 2h + 1)}{h}
\]
\[
= \lim_{h \to 0} 3x^2 + 3xh + h^2 - 4x - 2h + 1
= 3x^2 - 4x + 1
\]

2. (10 points) Use the definition of the derivative to find \( f'(x) \) for
\[
f(x) = \frac{3x - 2}{7x + 1}
\]

Solution:
\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{3(x + h) - 2}{7(x + h) + 1} - \frac{3x - 2}{7x + 1}}{h}
\]
\[
= \lim_{h \to 0} \frac{(3x + 3h - 2)(7x + 1) - (3x - 2)(7x + 7h + 1)}{h(7(x + h) + 1)(7x + 1)}
\]
\[
= \lim_{h \to 0} \frac{21x^2 + 3x + 21xh + 3h - 14x - 2 - 21x^2 - 21xh - 3x + 14x + 14h + 2}{h(7(x + h) + 1)(7x + 1)}
\]
\[
= \lim_{h \to 0} \frac{17h}{h(7(x + h) + 1)(7x + 1)}
\]
\[
= \frac{17}{(7x + 1)^2}
\]
3. (10 points) Use the definition of the derivative to find \( f'(x) \) for
\[
f(x) = \sqrt{4x + 9}
\]

Solution:
\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{4(x + h) + 9} - \sqrt{4x + 9}}{h} = \lim_{h \to 0} \frac{4h}{h\left(\sqrt{4(x + h) + 9} + \sqrt{4x + 9}\right)} = \frac{4}{2\sqrt{4x + 9}} = \frac{2}{\sqrt{4x + 9}}
\]

4. (20 points) Find \( f'(x) \) for
(a) \( f(x) = \sin(x)(1 - 2 \cos(x)) \)
Solution: \( f'(x) = \sin(x)(2\sin(x)) + \cos(x)(1 - 2 \cos(x)) \)

(b) \( f(x) = \frac{\tan(x)}{x^2 - 3x + 4} \)
Solution: \( f'(x) = \frac{(x^2 - 3x + 4) \sec^2(x) - \tan(x)(2x - 3)}{(x^2 - 3x + 4)^2} \)

(c) \( f(x) = e^{\sin(x)} + e^{\sec(x)} \)
Solution: \( f'(x) = e^{\sin(x)} \cos(x) + e^{\sec(x)} \sec(x) \tan(x) \)

(d) \( f(x) = \frac{e^{7x^2 + 3x - 9}(x^5 + 1)}{4^55^2x^2} \)
Solution:
\[
f'(x) = \frac{4^55^2x^2 \left[ e^{7x^2 + 3x - 9}x^4 + e^{7x^2 + 3x - 9}(14x + 3)(x^5 + 1) \right] - e^{7x^2 + 3x - 9}(x^7 + 1) \left[ 4^55^2x^4 4 \ln(5) + 4^55^2x \ln(4) \right]}{(4^55^2x^2)^2}
\]

5. (10 points) Use the squeeze theorem to show that \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \).
Solution: Done in class and in the text.

6. (10 points) Using the facts that \( \frac{d}{dx}(\sin(x)) = \cos(x) \) and \( \frac{d}{dx}(\cos(x)) = -\sin(x) \) derive the formula for \( \frac{d}{dx}(\cot(x)) \).
Solution: Done in class.
7. (10 points) Draw the graph of a function, \( f(x) \), with all of the following attributes.

(a) \( f(0) = 3, f(1) = -4, f(2) = 0, f(3) = 3 \) and \( f(5) = -1 \).
(b) \( f'(0) = -1, f'(1) = 1, f'(3) = 0, f'(4) = -2 \) and \( f'(6) = 2 \).
(c) \( f'(2) \) does not exist nor does it have a vertical tangent but \( f(x) \) is continuous at \( x = 2 \).
(d) \( f(x) \) has a vertical tangent at \( x = 5 \).

Solution: