1. (10 points) Use the definition of the derivative to find \( f'(x) \) for 
\[ f(x) = x^4 + 5x^2 - 2 \]

Solution:

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x + h)^4 + 5(x + h)^2 - 2 - (x^4 + 5x^2 - 2)}{h} \\
&= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 5x^2 + 10xh + 5h^2 - 2 - x^4 - 5x^2 + 2}{h} \\
&= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 + 10xh + 5h^2}{h} \\
&= \lim_{h \to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3 + 10x + 5h)}{h} \\
&= \lim_{h \to 0} 4x^3 + 6x^2h + 4xh^2 + h^3 + 10x + 5h \\
&= 4x^3 + 10x
\end{align*}
\]

2. (10 points) Use the definition of the derivative to find \( f'(x) \) for 
\[ f(x) = \frac{1}{3x + 1} \]

Solution:

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\frac{1}{3(x+h)+1} - \frac{1}{3x+1}}{h} \\
&= \lim_{h \to 0} \frac{\frac{3x+1 - 3(x+h) - 1}{(3(x+h)+1)(3x+1)}}{h} \\
&= \lim_{h \to 0} \frac{-3h}{(3(x+h)+1)(3x+1)} \\
&= \lim_{h \to 0} \frac{-3}{3(x+h) + 1)(3x + 1)} \\
&= \frac{-3}{(3x + 1)^2}
\end{align*}
\]
3. (10 points) Use the definition of the derivative to find \( f'(x) \) for \( f(x) = \frac{5}{\sqrt{17x - 3}} \)

Solution:

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{5}{\sqrt{17(x+h) - 3} - \sqrt{17x - 3}}
\]

\[
= 5 \lim_{h \to 0} \frac{\sqrt{17x - 3} - \sqrt{17(x+h) - 3}}{h(\sqrt{17(x+h) - 3} + \sqrt{17x - 3})}
\]

\[
= 5 \lim_{h \to 0} \frac{17x - 3 - 17x - 17h + 3}{-17h(\sqrt{17(x+h) - 3} + \sqrt{17x - 3})}
\]

\[
= 5 \lim_{h \to 0} \frac{17h}{\sqrt{17(x+h) - 3} + \sqrt{17x - 3}}
\]

\[
= \frac{-85}{2(17x - 3)^{3/2}}
\]

4. (20 points) Find \( f'(x) \) for

(a) \( f(x) = \tan(x)(e^x - 2^x) \)

Solution: \( f'(x) = \sec^2(x)(e^x - 2^x) + \tan(x)(e^x - 2^x \ln(2)) \)

(b) \( f(x) = \frac{\sin(x)}{4x^2} \)

Solution: \( f'(x) = \frac{4x^2 \cos(x) - 8x \sin(x)}{16x^4} = \frac{x \cos(x) - 2 \sin(x)}{4x^3} \)

(c) \( f(x) = \cos(e^{5x}) - \cot(e^{3x}) \)

Solution: \( f'(x) = -\sin(e^{5x})e^{5x}5 + \csc^2(e^{3x})e^{3x}3 \)

(d) \( f(x) = \left(x^2 - 3)^{2x}\right)^5 \sin(x) \)

Solution: \( f'(x) = 5 \left(x^2 - 3)^{2x}\right)^4 \frac{\sin(x)(\ln(2) + 2x^2) - (x^2 - 3)^2x \cos(x)}{\sin^2(x)} \)

5. (10 points) Use the fact that \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \) to show that \( \lim_{x \to 0} \frac{1 - \cos(x)}{x} = 0 \).

Solution: Done in class and it is in the text.

6. (10 points) Using the facts that \( \frac{d}{dx}(\sin(x)) = \cos(x) \) and \( \frac{d}{dx}(\cos(x)) = -\sin(x) \) derive the formula for \( \frac{d}{dx}(\csc(x)) \).

Solution: Done in class.
7. (10 points) Draw the graph of a function, \( f(x) \), with all of the following attributes.

(a) \( f(0) = -1, f(1) = 4, f(2) = 3, f(3) = 0 \) and \( f(5) = 1 \).
(b) \( f'(0) = 1, f'(1) = -2, f'(3) = 1, f'(4) = 0 \) and \( f'(6) = -2 \).
(c) \( f'(5) \) does not exist nor does it have a vertical tangent but \( f(x) \) is continuous at \( x = 5 \).
(d) \( f(x) \) has a vertical tangent at \( x = 2 \).

Solution:

![Graph of function f(x) with given attributes]