1. \((5\ \text{Points Each})\) Find the derivatives of the following functions, do not simplify your answers.

(a) \(f(x) = 3x^5 - 2x^2 + \pi x - \sqrt{x} + 7\)

Solution: 
\[f'(x) = 15x^4 - 4x + \pi - \frac{1}{2}x^{-2/3}.\]

(b) \(f(x) = x^4 \sin(x)\)

Solution: 
\[f'(x) = x^4 \cos(x) + 4x^3 \sin(x).\]

(c) \(f(x) = \frac{x + \tan(x)}{4x - 3}\)

Solution: 
\[f'(x) = \frac{(4x - 3)(1 + \sec^2(x)) - 4(x + \tan(x))}{(4x - 3)^2}.\]

(d) \(f(x) = \sin(4x^2 - 5x + 7)\)

Solution: 
\[f'(x) = (8x - 5) \cos(4x^2 - 5x + 7).\]

(e) \(f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}\}

Solution: 
\[f'(x) = \frac{1}{2} \left(x + (x + x^{1/2})^{1/2}\right)^{-1/2} \left(1 + \frac{1}{2} \left(x + x^{1/2}\right)^{-1/2}\right) \left(1 + \frac{1}{2}x^{-1/2}\right).\]

(f) \(f(x) = x^{\ln(x)}\)

Solution: 
\[f'(x) = x^{\ln(x)} \frac{2 \ln(x)}{x}.\]

(g) \(f(x) = \frac{4x^7 \sin(3x) \sqrt[3]{7x} \tan(x)}{x + \cos(\sin(x^3))}\)

Solution:
\[
f'(x) = \frac{4x^7 \sin(3x) \sqrt[3]{7x} \tan(x)}{x + \cos(\sin(x^3))} \left[\frac{7}{x} + \frac{3 \cos(3x)}{\sin(3x)} + 1 + \frac{1}{4} \left(1 + \frac{\sec^2(x)}{\tan(x)}\right) - \frac{1 - 3x^2 \sin(\sin(x^3)) \cos(x^3)}{x + \cos(\sin(x^3))}\right]
\]

2. \((10\ \text{Points})\) Find a formula for \(f^{(n)}(x)\) for the function \(f(x) = \ln(x)\).

Solution: 
\[f^{(n)}(x) = (-1)^{n+1} (n-1)! x^{-n}.\]

3. \((5\ \text{Points})\) Find \(\frac{dy}{dx}\) of \(\frac{x}{y} - y^3 + \cos(xy) = 3y\).

Solution: 
\[\frac{dy}{dx} = \frac{1 - y \sin(xy)}{3 + \frac{x}{y} + 3y^2 + x \sin(xy)}.\]

4. \((10\ \text{Points})\) Using differentials and linearization find an approximation to \(\sqrt{101.5}\).

Solution: 
\[f(x + \Delta x) \approx f(x) + f'(x) \Delta x = \sqrt{x} + \frac{\Delta x}{2 \sqrt{x}}, \text{ so } \sqrt{101.5} \approx 10 + \frac{1.5}{20} = 10.075\]

5. \((10\ \text{Points})\) A stone is dropped into a lake creating a circular ripple that travels outward at a speed of 20 inches per second. Find a formula for the rate in which the area of the circle is increasing.

Solution: 
Since \(A = \pi r^2\), \(\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 40\pi r\).

6. \((10\ \text{Points})\) Using \(\lim_{x \to 0} \frac{\sin(x)}{x} = 1\) and \(\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0\) and the definition of the derivative prove that \(\frac{d}{dx} (\sin(x)) = \cos(x)\).

Solution: 
This was done in class and it is in the text.
7. *(Extra Credit)* Do only one of the following.

(a) Derive the formula for $\frac{d}{dx} (\sec^{-1}(x))$.

**Solution:** $\frac{d}{dx} (\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2 - 1}}$.

\[
\begin{align*}
y & = \sec^{-1}(x) \\
x & = \sec(y) \\
1 & = \sec(y)\tan(y)y' \\
y' & = \frac{1}{\sec(y)\tan(y)} \\
& = \frac{\cos^2(y)}{\sin(y)} \\
& = \frac{1}{x^2 \sqrt{x^2 - 1}} \\
& = \frac{1}{x \sqrt{x^2 - 1}}
\end{align*}
\]

(b) Prove $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$.

**Solution:** This was done in class and it is in the text.

(c) Prove $\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$.

**Solution:** This was done in class and it is in the text.