1. (5 Points Each) Find the following limits using algebraic methods.

   (a) \( \lim_{{x \to -2}} \frac{3x^3 + 24}{x + 2} \)

   Solution:
   \[
   \lim_{{x \to -2}} \frac{3x^3 + 24}{x + 2} = \lim_{{x \to -2}} \frac{3(x^3 + 8)}{x + 2} = \lim_{{x \to -2}} \frac{3(x + 2)(x^2 - 2x + 4)}{x + 2} = \lim_{{x \to -2}} 3(x^2 - 2x + 4) = 36
   \]

   (b) \( \lim_{{x \to 1}} \frac{3x^3 + 24}{x + 2} \)

   Solution:
   \[
   \lim_{{x \to 1}} \frac{3x^3 + 24}{x + 2} = \frac{27}{3} = 9
   \]

   (c) \( \lim_{{x \to \frac{\pi}{2}^+}} \tan(x) \)

   Solution:
   \[
   \lim_{{x \to \frac{\pi}{2}^+}} \tan(x) = -\infty
   \]

   (d) \( \lim_{{x \to \frac{\pi}{2}^-}} \tan(x) \)

   Solution:
   \[
   \lim_{{x \to \frac{\pi}{2}^-}} \tan(x) = \infty
   \]

   (e) \( \lim_{{x \to \infty}} \sin(x) \)

   Solution:
   \[
   \lim_{{x \to \infty}} \sin(x) \text{ Does Not Exist}
   \]

2. (5 Points) Use the intermediate value theorem to prove that the function

   \[ f(x) = x^3 - \sin(x) + 2x + 3 \]

   has at least one real root.

   Solution: Since \( f(0) = 3 \) and \( f(-2) = -9 + \sin(2) < 0 \) we know by the intermediate value theorem that there is a root in the interval \([-2, 0]\).
3. (5 Points) Find the horizontal and vertical asymptotes of the function

\[ f(x) = \frac{\sqrt{3x^2 + x - 2}}{2x + 7} \]

**Solution:** The vertical asymptote will be when \( 2x + 7 = 0 \), that is, at \( x = -\frac{7}{2} \). As for the horizontal asymptotes,

\[
\lim_{x \to \infty} \frac{\sqrt{3x^2 + x - 2}}{2x + 7} = \lim_{x \to \infty} \frac{\sqrt{3 + 1/x - 2/x^2}}{2 + 7/x} = \frac{\sqrt{3}}{2}
\]

and

\[
\lim_{x \to -\infty} \frac{\sqrt{3x^2 + x - 2}}{2x + 7} = \lim_{x \to -\infty} \frac{\sqrt{3 + 1/x - 2/x^2}}{2 + 7/x} = -\frac{\sqrt{3}}{2}
\]

4. (10 Points) On the coordinate axes below draw the graph of a single function \( f(x) \) that has the following characteristics.

\[
\lim_{x \to 2^+} f(x) = 3 \quad \lim_{x \to 2^-} f(x) = -2 \quad f(2) \text{ DNE} \quad \lim_{x \to 7} f(x) = 3 \quad f(7) = -3
\]

\[
f'(0) = 1 \quad f'(-7) = -1 \quad f'(-9) = 1 \quad f'(-5) = 1
\]

The function must also have a vertical tangent line at \( x = 5 \) and its derivative does not exist at \( x = -3 \) but it is continuous at \( x = -3 \).

**Solution:**

[Graph image]
5. (5 Points) State the definition of the derivative of a function \( f(x) \).

Solution:

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

If this limit exists.

6. (10 Points Each) Use the definition of the derivative to find the derivative of the following functions.

(a) \( f(x) = 3x^2 - 2x + 1 \)

Solution:

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
      &= \lim_{h \to 0} \frac{3(x + h)^2 - 2(x + h) + 1 - (3x^2 - 2x + 1)}{h} \\
      &= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 1 - 3x^2 + 2x - 1}{h} \\
      &= \lim_{h \to 0} \frac{6xh + 3h^2 - 2h}{h} \\
      &= \lim_{h \to 0} \frac{h(6x + 3h - 2)}{h} \\
      &= \lim_{h \to 0} 6x + 3h - 2 \\
      &= 6x - 2 
\end{align*}
\]

(b) \( f(x) = \sqrt{3x^2 - 2} \)

Solution:

\[
\begin{align*}
  f'(x) &= \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \\
      &= \lim_{h \to 0} \frac{\sqrt{3(x + h)^2 - 2} - \sqrt{3x^2 - 2}}{h} \\
      &= \lim_{h \to 0} \frac{\sqrt{3(x + h)^2 - 2} - \sqrt{3x^2 - 2}}{h} \cdot \frac{\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2}}{\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2}} \\
      &= \lim_{h \to 0} \frac{3(x + h)^2 - 2 - (3x^2 - 2)}{h(\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2})} \\
      &= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 2 - (3x^2 - 2)}{h(\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2})} \\
      &= \lim_{h \to 0} \frac{h(6x + 3h)}{h(\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2})} \\
      &= \lim_{h \to 0} \frac{6x + 3h}{\sqrt{3(x + h)^2 - 2} + \sqrt{3x^2 - 2}} \\
      &= \frac{6x}{\sqrt{3x^2 - 2}} \\
      &= \frac{3x}{\sqrt{3x^2 - 2}} 
\end{align*}
\]
(c) \( f(x) = \frac{x}{x+1} \)

Solution:

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{x + h}{x+h+1} - \frac{x}{x+1} \\
&= \lim_{h \to 0} \frac{(x + h)(x + 1) - x(x + h + 1)}{h(x + 1)(x + h + 1)} \\
&= \lim_{h \to 0} \frac{x^2 + x + xh + h - x^2 - xh - x}{h(x + 1)(x + h + 1)} \\
&= \lim_{h \to 0} \frac{h}{h(x + 1)(x + h + 1)} \\
&= \frac{1}{(x + 1)(x + h + 1)} \\
&= \frac{1}{(x+1)^2}
\end{align*}
\]