1. (5 Points) Write the Mathematica command that defines the following function, \( f(x) = \frac{x^3 - \tan(x)}{\sec^2(x) + 5} \).

Solution: \( f[x_] := (x^3/5 - \tan[x])/(\sec[x]^2 + 5) \)

2. (5 Points) Write the Mathematica command that plots the above function on a domain of \([-5, 5]\). Draw the graph below.

Solution: \( \text{Plot}[f[x], \{x, -5, 5\}] \)

3. (5 Points) What is the Mathematica command to find the exact solutions of

\[1 + 4x - 19x^2 + 4x^3 + x^4 = 0\]

What are the exact solutions to this equation? Approximate these solutions to at least 7 decimal places.

Solution: \( \text{Solve}[1 + 4 x - 19 x^2 + 4 x^3 + x^4 == 0, x] \) followed by \( \text{N}[\%,10] \)

\( \{\{x \to \frac{1}{2} (-7 - 3 \sqrt{5})\}, \{x \to \frac{1}{2} (3 + \sqrt{5})\}, \{x \to \frac{1}{2} (-7 + 3 \sqrt{5})\}\} \)

\( \{\{x \to -6.854101966\}, \{x \to 0.3819660113\}, \{x \to 2.618033989\}, \{x \to 0.1458980338\}\} \)

4. (5 Points) What is the Mathematica command to find the following limit? Also, what is the limit?

\[ \lim_{x \to 0} \frac{\sqrt{x+4} - 2}{\sqrt{11x+9} - 3} \]

Solution: I accepted \( \text{Limit}[(\text{Sqrt}[x+4]-2)/(\text{Sqrt}[11x+9]-3), \ x \to 0] \) but this is only a one-sided limit so you should use the commands \( \text{Limit}[(\text{Sqrt}[x+4]-2)/(\text{Sqrt}[11x+9]-3), \ x \to 0, \ \text{Direction} \to 1] \) and \( \text{Limit}[(\text{Sqrt}[x+4]-2)/(\text{Sqrt}[11x+9]-3), \ x \to 0, \ \text{Direction} \to -1] \) to be sure that the two-sided limit exists. In both cases you find that the limit is \( \frac{3}{2} \).

5. (5 Points) Find the decimal approximation to \( \pi^{\pi^{1/\pi}} \) to at least 25 decimal places.

Solution: \( \text{N}[\pi^\pi^\pi, 50] \), gives 5.1964487580953794668045311046257048122159148249744.

6. (5 Points) What is the Mathematica command(s) to find the slope of the tangent line to \( f(x) = \cos(x)\sin(x) \) at \( x = \frac{\pi}{4} \). Find the exact slope and approximate it to at least 10 decimal places.

Solution: The commands are \( f[x_] := \cos[x]*\sin[x], f'[\pi/4] \) and \( \text{N}[\%, \ 20] \). The exact slope is

\[ 2^{-\frac{1}{4}} \left( -\frac{1}{\sqrt{2}} - \frac{\text{Log}[2]}{2\sqrt{2}} \right) \]

which approximates to \(-0.74522071602462836376\).