1. Find the following limit using algebraic methods,

\[ \lim_{x \to \infty} \sqrt{x^2 - 3x + 11 - x} \]

**Solution:**

\[
\begin{align*}
\lim_{x \to \infty} \sqrt{x^2 - 3x + 11 - x} &= \lim_{x \to \infty} \left( \sqrt{x^2 - 3x + 11} - x \right) \frac{\sqrt{x^2 - 3x + 11} + x}{\sqrt{x^2 - 3x + 11} - x} \\
&= \lim_{x \to \infty} \frac{x^2 - 3x + 11 - x^2}{\sqrt{x^2 - 3x + 11} + x} \\
&= \lim_{x \to \infty} \frac{-3x + 11}{\sqrt{x^2 - 3x + 11} + x} \\
&= \lim_{x \to \infty} \frac{-3 + \frac{11}{x}}{\sqrt{1 + \frac{3}{x} + \frac{11}{x^2}} + 1} \\
&= \lim_{x \to \infty} \frac{-3 + \frac{11}{x}}{\sqrt{1 + \frac{3}{x} + \frac{11}{x^2}} + 1} \\
&= \frac{-3}{2}
\end{align*}
\]

2. State why the following function is continuous on its domain and find its domain,

\[ f(x) = \frac{x^4 + \sin(x)}{e^x \sqrt{x^2 - 7}} \]

**Solution:** Since trigonometric functions, root functions, exponential functions and polynomials are continuous on their domains and this function is produced by sums products and quotients of such functions it too is continuous on its domain. Since \( \sin(x) \) has a domain of all real numbers as do polynomials and exponential functions (like \( e^x \)) the domain will be whenever \( x^2 - 7 > 0 \). This inequality is true when \( x > \sqrt{7} \) and when \( x < -\sqrt{7} \). So the domain is \((-\infty, -\sqrt{7}) \cup (\sqrt{7}, \infty)\).