A graph of the relation $x^3 - 2xy^2 = 10$ is given below. Find the equations of the tangent lines to this curve when $x = -1$.

Solution:

First find the slope of any tangent line $y'$ using implicit differentiation.

\[
\begin{align*}
x^3 - 2xy^2 &= 10 \\
3x^2 - 4xyy' - 2y^2 &= 0 \\
-4xyy' &= 2y^2 - 3x^2 \\
y' &= \frac{2y^2 - 3x^2}{-4xy}
\end{align*}
\]

Now find the points on the curve where $x = -1$. When $x = -1$ our equation reduces to $-1 + 2y^2 = 10$ which has the solutions $y = \pm \sqrt{\frac{11}{2}}$.

So when $x = -1$ and $y = \pm \sqrt{\frac{11}{2}}$ the slope of the tangent line is

\[
y' = \frac{2\left(-\sqrt{\frac{11}{2}}\right)^2 - 3(-1)^2}{-4(-1)\sqrt{\frac{11}{2}}} \approx -0.852802865422442
\]

So the equation of the tangent line there is

\[
y = -0.852802865422442(x + 1) - \sqrt{\frac{11}{2}}
\]

When $x = -1$ and $y = \sqrt{\frac{11}{2}}$ the slope of the tangent line is

\[
y' = \frac{2\left(-\sqrt{\frac{11}{2}}\right)^2 - 3(-1)^2}{-4(-1)\left(-\sqrt{\frac{11}{2}}\right)} \approx 0.852802865422442
\]

So the equation of the tangent line there is

\[
y = 0.852802865422442(x + 1) + \sqrt{\frac{11}{2}}
\]

\[
y = 3.19801 + 0.852803x
\]