1. (20 points) This exercise will be dealing with the function \( f(x) = x \sin(x) \) on the interval \([0, 5]\).

(a) Using 5 rectangles find the right hand Riemann sum that approximates the net area of \( f(x) \) on the interval \([0, 5]\). Your answer must be correct to at least three decimal places.

\[ \text{Solution:} \quad A \approx \sin(1) + 2 \sin(2) + 3 \sin(3) + 4 \sin(4) + 5 \sin(5) = -4.738405492 \ldots \]

(b) Using limit and summation notation, write an expression for the exact area of \( f(x) \) on the interval \([0, 5]\). Do not evaluate the sum or the limit.

\[ \text{Solution:} \quad \lim_{n \to \infty} \sum_{i=1}^{n} \frac{5i}{n} \sin \left( \frac{5i}{n} \right) \frac{5}{n} \]

(c) Using the sum and limit commands in Maple, write the command that will find the exact area of \( f(x) \) on the interval \([0, 5]\), using 5 rectangles.

\[ \text{Solution:} \quad \text{RiemannSum}(x \sin(x), x=0..5, \text{method=right}, \text{output=plot}, \text{partition}=5); \]

2. (15 points) This exercise will be dealing with the function \( f(x) = x + x^2 \) on the interval \([1, 2]\).

(a) Using 5 rectangles find the right hand Riemann sum that approximates the net area of \( f(x) \) on the interval \([1, 2]\). Your answer must be correct to at least three decimal places.

\[ \text{Solution:} \quad A \approx 1.2 + 2.4 + (1.4 + 1.4^2) + 0.2 + (1.6 + 1.6^2) + 0.2 + (1.8 + 1.8^2) + 0.2 + (2 + 2^2) = 4.24 \]

(b) Using limit and summation notation, write an expression for the exact area of \( f(x) \) on the interval \([1, 2]\). Do not evaluate the sum or the limit.

\[ \text{Solution:} \quad \lim_{n \to \infty} \sum_{i=1}^{n} \left( 1 + \frac{i}{n} \right) + \left( 1 + \frac{i}{n} \right)^2 \frac{1}{n} \]

(c) Using the formulas for sums and limits evaluate the expression for the exact area of \( f(x) \) on the interval \([1, 2]\). You may not use the Fundamental Theorem of Calculus.

\[ \text{Solution:} \quad \lim_{n \to \infty} \sum_{i=1}^{n} \left( 1 + \frac{i}{n} \right) + \left( 1 + \frac{i}{n} \right)^2 \frac{1}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{3}{n} + \frac{1}{n^2} \right) \frac{1}{n} = \lim_{n \to \infty} \frac{1}{n} \left( 2n + \frac{3}{n} \sum_{i=1}^{n} i + \frac{1}{n^2} \sum_{i=1}^{n} i^2 \right) = \lim_{n \to \infty} \frac{1}{n} \left( 2n + \frac{3}{n} \cdot \frac{n(n+1)}{2} + \frac{1}{n^2} \frac{n(n+1)(2n+1)}{6} \right) = \lim_{n \to \infty} \frac{23n}{6} + \frac{2}{n} + \frac{1}{6n^2} = \frac{23}{6} = 3.833333333 \ldots \]

3. (5 points each) Using your integral rules and the Fundamental Theorem of Calculus evaluate the following.

(a) \[ \int x^3 - 3x^2 + x - 2 \, dx = \frac{x^4}{4} - x^3 + \frac{x^2}{2} - 2x + C \]

(b) \[ \int \tan(x) \, dx = -\ln(\cos(x)) + C \]

(c) \[ \int_{1}^{2} x^3 - 3x \, dx = -3x^2 + \frac{x^4}{4} + \frac{19}{4} = 727.8223285 \ldots \]

(d) \[ \frac{d}{dx} \int_{1}^{x^2} t \sin(t) \tan(t) \, dt = 2x^3 \sin(x^2) \tan(x^2) \]
4. (15 points) Find the area of the region bounded by the curves \( f(x) = -x^2 - x + 3 \) and \( g(x) = x^2 + 2 \).

**Solution:** 
\[
A = \int_{-1}^{1/2} (-x^2 - x + 3) - (x^2 + 2) \, dx = \frac{9}{8} = 1.125
\]

5. (15 points) Setup but do not evaluate the integral that will find the volume of the region bounded by \( f(x) = -x^2 - x + 3 \) and \( g(x) = x^2 + 2 \) rotated about the \( x \)-axis.

**Solution:** 
\[
V = \int_{-1}^{1/2} \pi \left( -x^2 - x + 3 \right)^2 - \pi \left( x^2 + 2 \right)^2 \, dx
\]

6. (15 points) Setup but do not evaluate the integral that will find the volume of the region bounded by \( y = \sin(x) \), \( y = 0 \), \( x = 0 \) and \( x = \pi \) rotated about the \( y \)-axis.

**Solution:** 
\[
V = \int_{0}^{\pi} 2\pi x \sin(x) \, dx
\]

7. (10 points) Find the volume of an object that has a circular base of radius 1 and each perpendicular cross section along one diameter is an equilateral triangle.

**Solution:** Place the circular base at the origin and the diameter of interest on the \( x \)-axis. Then the length of the side of the equilateral triangle at position \( x \) is \( L = 2\sqrt{1 - x^2} \). Also, the cross section has area \( A = \frac{1}{2} LH \) where \( H = \frac{\sqrt{3}}{2} L \), giving \( A = \sqrt{3} (1 - x^2) \). So our volume is
\[
\int_{-1}^{1} \sqrt{3} (1 - x^2) \, dx = \frac{4\sqrt{3}}{3}
\]