Name:__________________________________________________________

Write all of your responses on the exam paper. If you need extra space, please use the backs of the exam pages. Make sure that you show all of your work. Answers without supporting work will receive no credit.

1. (15 Points) Find the derivative of the following function.

\[ f(x) = \int_{\sqrt{3}}^{x^3} \sqrt{t} \sin(t) \, dt \]

2. (15 Points) Find the following integral.

\[ \int \frac{\sin(x)}{1 + \cos^2(x)} \, dx \]
3. (15 Points) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = x^2$ and $x = y^2$ about the line $y = -1$.

4. (15 Points) Find the following integral.

$$\int e^{2x} \sin(3x) \, dx$$
5. (20 Points) Find the following integral.

$$\int \frac{x^2}{\sqrt{4x - x^2}} \, dx$$

6. (15 Points) Find the length of the curve $y = \ln(\cos(x))$ for $0 \leq x \leq \pi/3$. 
7. (15 Points) Use Euler’s method with step size 0.1 to estimate \( y(1.4) \) where \( y(x) \) is the solution of the initial-value problem \( y' = x - xy \) and \( y(1) = 0 \).

8. (20 Points) Find the solution of the differential equation

\[
x \cos(x) = (2y + e^{3y}) y'
\]

with the initial condition \( y(0) = 0 \). You do not need to solve the final answer for \( y \) but put all \( y \) terms on one side of the equal sign and all \( x \) terms and constants on the other side of the equal sign.
9. (15 Points) Test the following series for convergence or divergence

\[ \sum_{n=1}^{\infty} \frac{3^n n^2}{n!} \]

10. (15 Points) Test the following series for convergence or divergence

\[ \sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5} \]
11. (20 Points) Find the radius and interval of convergence of the series

\[ \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n3^n} \]

12. (20 Points) Find the Taylor series for \( f(x) = x^{-2} \) centered at \( a = 1 \). Write your final answer in summation form.
13. *(Extra Credit: 10 Points)* Gabriel’s horn is defined as the surface generated by revolving the curve $y = 1/x$, $1 \leq x < \infty$ around the $x$-axis. Find the volume and surface area of the horn and then explain why this is an object that you can fill with paint but you cannot paint it. An image of the horn is below, you are looking down the bell of the horn. Note that the horn does go on forever.