Name: ________________________________

Write all of your responses on the exam paper. If you need extra space, please use the backs of the exam pages. Make sure that you show all of your work. Answers without supporting work will receive no credit.

1. (15 Points Each): Find the following integrals.

(a) \[ \int 3 + x\sqrt{x} \, dx \]

(b) \[ \int \frac{x^2}{\sqrt{3x^3 - 4}} \, dx \]
(c) \[ \int_{1}^{\sqrt{3}} \tan^{-1}(1/x) \, dx \]

(d) \[ \int \sin^5(x) \cos^4(x) \, dx \]
(c) \[ \int \frac{1}{\sqrt{x^2 - 2x + 5}} \, dx \]

2. \textit{(15 Points)}: Find the volume of the object that has a circular base of radius 1 and perpendicular cross sections to a diameter are triangles whose altitude is equal to the length of its base.
3. (15 Points): Write the integral that will find the arc length of \( f(x) = x^3 \) over the interval \( 0 \leq x \leq 1 \), but do not evaluate the integral. Use Simpson’s Rule with \( n = 8 \) subdivisions to approximate this integral and hence approximate the arc length. Finally, write the Maple command that will evaluate the integral.

4. (15 Points): Find the solution to the differential equation \( \frac{dy}{dx} = (y^2 + 1) \cos(x) \) that satisfies the initial condition of \( y(0) = 1 \).
5. (15 Points): Scientists can determine the age of ancient objects by a method called radiocarbon dating. The bombardment of the upper atmosphere by cosmic rays converts nitrogen to a radioactive isotope of carbon, $^{14}$C, with a half-life of about 5730 years. Vegetation absorbs carbon dioxide through the atmosphere and animal life assimilates $^{14}$C through the food chain. When a plant or animal dies, it stops replacing its carbon and the amount of $^{14}$C begins to decrease through radioactive decay. Therefore, the level of radioactivity must also decay exponentially. A parchment fragment was discovered that had about 71% as much $^{14}$C radioactivity as does plant material on Earth today. Estimate the age of the parchment.

6. (15 Points): Find the sum of the series $\sum_{n=4}^{\infty} \frac{5}{n^2 + n - 6}$. 
7. (15 Points): Determine if the series \( \sum_{n=2}^{\infty} \frac{\ln(n)}{n^2} \) converges or diverges.

8. (15 Points): Determine if the series \( \sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^2} \) is absolutely convergent, conditionally convergent or divergent.
9. \textit{(15 Points)}: Find the Maclaurin series for $\ln(8 - 3x)$ and determine its interval of convergence.
10. (15 Points): Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $x = 10$ and determine its interval of convergence.