1. (15 Points Each) Find the following integrals.

(a) \( \int x^2 \sin(x) \, dx \)

Solution: \( \int x^2 \sin(x) \, dx = -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C \)

(b) \( \int \tan^6(x) \sec^4(x) \, dx \)

Solution: \( \int \tan^6(x) \sec^4(x) \, dx = \frac{1}{9} \tan^9(x) + \frac{1}{7} \tan^7(x) + C \)

(c) \( \int \frac{x}{\sqrt{x^2 + x + 1}} \, dx \)

Solution: \( \int \frac{x}{\sqrt{x^2 + x + 1}} \, dx = \sqrt{x^2 + x + 1} - \frac{1}{2} \ln \left| \frac{2}{\sqrt{3}} \sqrt{x^2 + x + 1} + \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right) \right| + C \)

(d) \( \int \frac{2x - 1}{x^3 - 4x^2 - 21x} \, dx \)

Solution: \( \int \frac{2x - 1}{x^3 - 4x^2 - 21x} \, dx = \frac{1}{21} \ln|x| + \frac{13}{70} \ln|x - 7| - \frac{7}{30} \ln|x + 3| + C \)

(e) \( \int \frac{1 + \sin(x)}{1 - \sin(x)} \, dx \)

Solution: \( \int \frac{1 + \sin(x)}{1 - \sin(x)} \, dx = 2 \tan(x) + 2 \sec(x) - x + C \)

(f) \( \int \frac{\sin(x) \cos(x)}{\sin^4(x) + \cos^4(x)} \, dx \)

Solution: \( \int \frac{\sin(x) \cos(x)}{\sin^4(x) + \cos^4(x)} \, dx = \frac{1}{2} \tan^{-1}(2 \sin^2(x) - 1) + C \)

2. (20 Points) Answer the following set of questions.

(a) Find the Trapezoidal Rule approximation to \( \int_0^2 \sin(x^2) \, dx \) using 4 divisions, round your answer to at least six decimal places.

Solution: \( T_4 = 0.7442734466 \).

(b) Find the Simpson’s Rule approximation to \( \int_0^2 \sin(x^2) \, dx \) using 4 divisions, round your answer to at least six decimal places.

Solution: \( S_4 = 0.8380080165 \).
(c) Find the error bounds for the two approximations above, the graphs of the second derivative and the fourth derivative of the integrand on the interval [0, 2] are below. The first image is the second derivative, and the second image is the fourth derivative.

Solution: $|E_T| \leq \frac{K(b-a)^3}{(b-a)^2} = \frac{12 \cdot 2^3}{12 \cdot 4^2} = \frac{1}{2} = 0.5$ and $|E_S| \leq \frac{K(b-a)^5}{(b-a)^4} = \frac{170 \cdot 2^5}{180 \cdot 4^4} = 0.11805555555\ldots$

(d) Write the Maple commands that will find the Trapezoidal Rule and Simpsons Rule approximations to the integral $\int_0^2 \sin(x^2) \, dx$ using 4 divisions.

Solution:

```maple
with(Student[Calculus1]):
ApproximateInt(sin(x^2), x=0..2, method=trapezoid, partition=4);
ApproximateInt(sin(x^2), x=0..2, method=simpson, partition=4);
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