Find the volume of the object obtained by rotating the region bounded by \( y = \frac{1}{\sqrt{x}} \), \( y = x \) and \( x = \frac{1}{2} \) about the \( x \)-axis.

**Solution:** First find the bounds of this region. \( \frac{1}{\sqrt{x}} = x \) gives

\[
\begin{align*}
\frac{1}{\sqrt{x}} &= x \\
\frac{1}{x} &= x^2 \\
x^3 &= 1
\end{align*}
\]

So \( x = 1 \). Hence the volume is

\[
\int_{1/2}^{1} \pi \left( \frac{1}{\sqrt{x}} \right)^2 - \pi x^2 \, dx = \pi \int_{1/2}^{1} \frac{1}{x} - x^2 \, dx
\]

\[
= \pi \left[ \ln |x| - x^3 \right]_{1/2}^{1}
\]

\[
= \pi \left[ \left( \ln(1) - \frac{1}{3} \right) - \left( \ln(1/2) - \frac{1}{24} \right) \right]
\]

\[
= \pi \left[ -\ln(1/2) - \frac{7}{24} \right]
\]

\[
\approx 1.2612882330065791026
\]