**Exam #3 Key**

**Concepts:** For each of the following, give a one or two sentence explanation that best answers the question.

1. *(4 Points)* State the Central Limit Theorem? *Pages 304–305*

2. *(3 Points)* What is a Type I error? *Page 373*

3. *(3 Points)* What is a Type II error? *Page 373*

4. *(5 Points)* List all of the elements of a Hypothesis Test. *Page 374*

**Calculations:**

1. *(15 Points)* The following data set is a sample of 16 similar carpets and the variable of interest is durability. We can assume that the population of all durability measurements of this type of carpet is normally distributed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>N*</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durability</td>
<td>16</td>
<td>0</td>
<td>13.79</td>
<td>1.14</td>
<td>4.54</td>
<td>7.03</td>
<td>10.58</td>
<td>12.95</td>
<td>17.24</td>
</tr>
</tbody>
</table>

(a) Construct a 95% confidence interval for the durability.

**Solution:** $13.79 \pm 2.131 \cdot \frac{4.54}{\sqrt{16}} = [11.371315, 16.208685]

(b) Do a complete hypothesis test at the $\alpha = 0.10$ level to determine if the true population mean of the carpet durability is below 15. Include the $p$-value of this test.

**Solution:**

i. $H_0 : \mu = 15$, $H_a : \mu < 15$.

ii. Test Statistic: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

iii. Rejection Region: $t < -1.341$

iv. Assumptions:

A. A random sample is selected from the target population.

B. The population distribution is approximately normal.

v. Experiment: $t = \frac{13.79 - 15}{4.54/\sqrt{16}} = -1.066079295$

vi. Conclusion: Do not reject $H_0$.

vii. P-Value: From the table $p > 0.1$, from a calculator $p = 0.1516236523$

(c) Did we need to assume that the population of all durability measurements of this type of carpet is normally distributed?

**Solution:** Yes, since our sample size is small we need the assumption that all durability measurements of this type of carpet is normally distributed.

2. *(10 Points)* In a pilot study to determine the proportion of blue eyed students at Salisbury University we took a sample of 126 students and found that 37 of them had blue eyes. How big must our sample size be in order to construct a 99% confidence interval for the proportion of blue eyed students at Salisbury University of length no more than 0.05?

**Solution:**

$$n = \frac{2.575^2 \left(\frac{37}{126} \cdot \frac{89}{126}\right)}{0.05^2} = 2200.518833 \text{ so } n = 2201.$$

3. *(10 Points)* In a sample of 68 power drills we have the following sample statistics on the torque produced by these drills.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>N*</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque</td>
<td>68</td>
<td>0</td>
<td>21.265</td>
<td>0.779</td>
<td>6.422</td>
<td>10.000</td>
<td>16.000</td>
<td>20.000</td>
<td>24.750</td>
</tr>
</tbody>
</table>

(a) Construct a 99% confidence interval for the torque.

**Solution:** $21.265 \pm 2.575 \cdot 6.422/\sqrt{68} = [19.25963663, 23.27036337]$
(b) Do a complete hypothesis test at the $\alpha = 0.05$ level to determine if the true population mean of the torque is different than 23. Include the $p$-value of this test.

**Solution:**

i. $H_0 : \mu = 23$, $H_a : \mu \neq 23$.

ii. Test Statistic: $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$

iii. Rejection Region: $z < -1.96$ or $z > 1.96$

iv. Assumptions:
   A. A random sample is selected from the target population.
   B. The sample size is large.

v. Experiment:
   $z = \frac{21.265 - 23}{0.422/\sqrt{68}} = -2.227838138$

vi. Conclusion: Reject $H_0$.

vii. $P$-Value: From the table $p = 2(0.5 - 0.4871) = 0.0258$, from a calculator $p = 0.0258912236$

4. (10 Points) In the following sample we took 12 sales measurements from Walmart stores on the eastern shore. Does this data support the claim that the population standard deviation is less than 20? Do a hypothesis test at the $\alpha = 0.05$ level to answer the question and you may assume that all needed assumptions are met.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>N*</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>12</td>
<td>0</td>
<td>49.92</td>
<td>4.26</td>
<td>14.76</td>
<td>29.00</td>
<td>35.25</td>
<td>50.50</td>
<td>65.25</td>
<td>71.00</td>
</tr>
</tbody>
</table>

**Solution:**

(a) $H_0 : \sigma^2 = 400$, $H_a : \sigma^2 < 400$.

(b) Test Statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

(c) Rejection Region: $\chi^2 < 4.57481$

(d) Assumptions:
   i. A random sample is selected from the target population.
   ii. The population distribution is approximately normal.

(e) Experiment: $\chi^2 = \frac{11.14.76^2}{400} = 5.991084$

(f) Conclusion: Do not reject $H_0$.

5. (15 Points) Independent random samples of sizes $n_1 = 312$ and $n_2 = 432$ were selected from two binomial populations. The samples produced 231 and 300 successes, respectively.

(a) Construct a 90% confidence interval for $p_1 - p_2$.

**Solution:** $\frac{231}{312} - \frac{300}{432} \pm 1.645 \sqrt{\frac{231}{312} \frac{311}{312} + \frac{300}{432} \frac{431}{432}} = [-0.0087980087, 0.106783505]$\]

(b) Do a complete hypothesis test at the $\alpha = 0.05$ level to determine if the true population proportions are different. Include the $p$-value of this test.

**Solution:**

i. $H_0 : p_1 - p_2 = 0$, $H_a : p_1 - p_2 \neq 0$.

ii. Test Statistic: $z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(1/n_1 + 1/n_2)}}$

iii. Rejection Region: $z < -1.96$ or $z > 1.96$

iv. Assumptions:
   A. Random samples selected from the target populations.
   B. Both $n_1$ and $n_2$ are large. They both pass the $\hat{p} \pm 3\sigma_{\hat{p}}$ test.

v. Experiment: $z = \frac{231 - 300}{\sqrt{\frac{231}{312} + \frac{300}{432}}} = 1.367921523$

vi. Conclusion: Do not reject $H_0$.

vii. $P$-Value: From the table $p = 2(0.5 - 0.4147) = 0.1706$, from a calculator $p = 0.1713367571$
6. (10 Points) The following descriptive statistics are from before and after test scores given to 17 participants in a training program. Each of the 17 were given a test before the program and a test after they completed the program. We then took the after score and subtracted the before score for each participant. Test to see if the completion of the program made a significant improvement in the participants performance. Use $\alpha = 0.05$ for this test and you may assume that all needed assumptions are met.

### Variable N N* Mean SE Mean StDev
Aft. - Bef. 17 0 17.47 3.68 15.18

**Solution:**

(a) $H_0 : \mu_D = 0$, $H_a : \mu_D > 0$.

(b) Test Statistic: $t = \frac{\bar{x}_D - D_0}{s_D/\sqrt{n_D}}$

(c) Rejection Region: $t > 1.746$

(d) Assumptions:
   i. Random samples selected from the target populations.
   ii. The population of differences is approximately normal.

(e) Experiment: $t = \frac{17.47}{15.18/\sqrt{17}} = 4.745102456$

(f) Conclusion: Reject $H_0$.

7. (10 Points) In a hypothesis test of $\mu > 25$ at the $\alpha = 0.05$ level we took a sample of 64 measurements and got a mean of $\bar{x} = 27.2$ and a standard deviation of $s = 15.687$. The population in question is uniformly distributed. If the true population mean was $\mu = 30$ what was the probability we made a Type II error?

**Solution:** First, $\bar{x}_0 = \mu_0 + z_\alpha \frac{s}{\sqrt{n}} = 25 + 1.645 \frac{15.687}{\sqrt{64}} = 28.22563938$ which gives $z = \frac{\bar{x}_0 - \mu_0}{s/\sqrt{n}} = \frac{28.22563938 - 30}{15.687/\sqrt{64}} = -0.904882068$. From the table we get $\beta = 0.1841$ and using a calculator we get $\beta = 0.182763902$.

8. (15 Points) We tested a manufacturing process using two different energy sources. First do a hypothesis test to determine if the two population variances are equal (use $\alpha = 0.05$) then do the appropriate test to see if the population means differ (again use $\alpha = 0.05$). State the assumptions that we need to make for each test and you may assume that these hold for our respective populations.

### Energy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
<th>N</th>
<th>N*</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process 1 A</td>
<td>16</td>
<td>0</td>
<td>48.333</td>
<td>0.494</td>
<td>1.915</td>
<td>45.000</td>
<td>46.000</td>
<td>48.000</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>15</td>
<td>0</td>
<td>34.188</td>
<td>0.390</td>
<td>1.559</td>
<td>31.000</td>
<td>33.000</td>
<td>34.000</td>
<td></td>
</tr>
</tbody>
</table>

**Solution:** First we test the population variances,

(a) $H_0 : \sigma_1^2 = \sigma_2^2$, $H_a : \sigma_1^2 \neq \sigma_2^2$.

(b) Test Statistic: $F = \frac{\sigma_1^2}{\sigma_2^2}$

(c) Rejection Region: $F > F_{\alpha/2} = 2.95$

(d) Assumptions:
   i. Random samples selected from the target populations.
   ii. The populations are normally distributed.

(e) Experiment: $F = \frac{1.9152}{1.559} = 1.508847426$

(f) Conclusion: Do not reject $H_0$.

Since we did not reject $H_0$ in the variance test we may assume that the two population variances are equal. So in our test of the difference of the means we may use a pooled $t$ test.

(a) $H_0 : \mu_1 - \mu_2 = 0$, $H_a : \mu_1 - \mu_2 \neq 0$. 


(b) Test Statistic: \[ t = \frac{\mu_1 - \mu_2}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \] where \[ s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}. \]

(c) Rejection Region: \( t < -2.045 \) or \( t > 2.045 \)

(d) Assumptions:
   i. Random samples selected from the target populations.
   ii. The populations are normally distributed.
   iii. \( \sigma_1^2 = \sigma_2^2 \)

(e) Experiment: \[ s_p^2 = \frac{15.915^2 + 14.159^2}{29} = 3.070176172 \] and \( t = \frac{14.145}{\sqrt{3.070176172 \left( \frac{1}{16} + \frac{1}{15} \right)}} = 22.46187869 \)

(f) Conclusion: Reject \( H_0 \).