1. Find the volume of the region bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$. Several graphs of the surfaces are below.

**Solution:** Note that the intersection between these two surfaces is a circle. Hence we will be integrating over a circle and so it would be advantageous for us to use polar coordinates. In doing so our volume becomes

$$
\iint_D (4 - x^2 - y^2) - (3x^2 + 3y^2) \, dA = \int_0^{2\pi} \int_0^1 ((4 - r^2) - 3r^2) \, r \, dr \, d\theta
$$

$$
= 4 \int_0^{2\pi} \int_0^1 r - r^3 \, dr \, d\theta
$$

$$
= 4 \int_0^{2\pi} \left[ \frac{r^2}{2} - \frac{r^4}{4} \right]_0^1 \, d\theta
$$

$$
= 4 \int_0^{2\pi} \frac{1}{4} \, d\theta
$$

$$
= \int_0^{2\pi} \, d\theta
$$

$$
= 2\pi
$$