Final Exam

Write all of your responses on the extra paper provided. Make sure that you show all of your work. Answers without supporting work will receive no credit. Remember that there is no sharing of calculation devices on this exam. Do one and only one problem from each part. Each part will be worth 25 points.

Part 1:
1. Find the scalar and vector projections of \((2, 5, 9)\) onto \((-1, 3, -5)\).
2. Find the angle between the vectors \((2, 5, 9)\) and \((-1, 3, -5)\).

Part 2:
1. Find an equation for the plane that passes through the points \((2, 5, -2)\), \((3, -7, 1)\) and \((4, 2, -7)\).
2. Find the parametric equations for the line passing through the point \((1, 2, 3)\) and orthogonal to the plane passing through the points \((0, 0, 0)\), \((2, -4, 1)\) and \((1, 1, -5)\).

Part 3:
1. Find an equation of the tangent line to the curve \(r(t) = \langle \cos(t), \sin(t) \cos(t), \ln(t) \rangle\) at \((-1, 0, \ln(\pi))\).
2. Find the length of the curve \(r(t) = \langle t^2, \sin(t) - t \cos(t), \cos(t) + t \sin(t) \rangle\) for \(0 \leq t \leq \pi\).

Part 4:
1. Find an equation for the osculating plane to the curve \(r(t) = \langle t^2, t^3, t^5 \rangle\) at \((4, -8, -32)\).
2. Find the curvature of \(r(t) = \langle t^2, t^3, t^5 \rangle\) at \((4, -8, -32)\).

Part 5:
1. Find \(\frac{\partial z}{\partial x}\) and \(\frac{\partial z}{\partial y}\) of \(\ln(x + yz) = 1 + xy^2z^3\).
2. Find the local maximums and minimums of \(f(x, y) = e^{\sin(x)} \cos(y)\) on the domain \([-5, 5] \times [-5, 5]\). A few images of this surface are below.
Part 6:

1. Find
\[
\lim_{(x,y) \to (2,0)} \frac{xy - 2y}{x^2 + y^2 - 4x + 4}
\]

2. Find the directional derivative of \( f(x, y, z) = z^3 - x^2y \) at \((1, 6, 2)\) in the direction of \((3, 5, 12)\)

Part 7:

1. Find
\[
\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) \, dy \, dx
\]

2. Find the surface area of the part of the surface \( z = x + y^2 \) that lies above the triangle with vertices \((0, 0), (1, 1)\) and \((0, 1)\).

Part 8:

1. Find the volume of the solid that lies within the sphere \( x^2 + y^2 + z^2 = 4 \), above the \( xy\)-plane and below the cone \( z = \sqrt{x^2 + y^2} \). A few images of this volume are below.

2. Use the transformation \( x = 7u + 2v, y = u - 3v \) to find the integral
\[
\int \int_R x^2 + y^2 \, dA
\]

where \( R \) is region bounded by the points \((0, 0), (21, 3), (10, -15)\) and \((31, -12)\).