Find the volume bounded by the surface \( f(x, y) = \sin(x^3)\cos(x^3) \), the surface \( y = x^2 \) and the planes \( x = 0, x = 1, y = 0 \) and \( z = 0 \).

**Solution:** Since the first integral is with respect to \( y \) and the function has only \( x \)'s in it, the first integral is of a constant.

\[
\int_0^1 \int_0^{x^2} \sin(x^3)\cos(x^3) \, dy \, dx = \int_0^1 x^2 \sin(x^3)\cos(x^3) \, dx
\]

Now we can use \( u \) substitution. Let \( u = \sin(x^3) \), then \( du = 3x^2\cos(x^3) \, dx \) and \( dx = \frac{du}{3x^2\cos(x^3)} \). So,

\[
\int_0^1 x^2 \sin(x^3)\cos(x^3) \, dx = \int_0^1 x^2u \cos(x^3) \frac{du}{3x^2\cos(x^3)}
\]

\[
= \int_0^1 u \frac{du}{3}
\]

\[
= \frac{1}{3} \left[ \frac{1}{2} u^2 \right]
\]

\[
= \frac{1}{6} \left[ \sin^2(x^3) \right]_0^1
\]

\[
= \frac{1}{6} \left[ \sin^2(1) - \sin^2(0) \right]
\]

\[
= \frac{1}{6} \sin^2(1)
\]

\[
\approx 0.118012236379
\]