1. (10 Points) Let \( R \) be a relation on \( N \times N \) defined as \( R = \{(x, y) | y - x \text{ is even}\} \). Prove or disprove: \( R \) is an equivalence relation.

**Solution:** To show that \( R \) is an equivalence relation we must show that \( R \) is reflexive, symmetric and transitive. To show that \( R \) is reflexive let \( a \in N \). Since \( a - a = 0 \), which is even, we have \((a, a) \in R\) and hence \( R \) is reflexive. To show that \( R \) is symmetric let \((a, b) \in R\). So \( b - a \) is even and hence so is \( a - b = -(b - a) \), so \((b, a) \in R\) and \( R \) is symmetric. Finally, to show that \( R \) is transitive let \((a, b) \in R\) and \((b, c) \in R\). Then both \( b - a \) and \( c - b \) are even and hence so is their sum \((b - a) + (c - b) = c - a\). Thus, \((a, c) \in R\) and \( R \) is transitive and therefore \( R \) is an equivalence relation.

2. (15 Points) Consider the following deterministic finite automaton.

(a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

i. \( aabab \)
   **Solution:** SQMNPP — Accepted

ii. \( baabab \)
   **Solution:** SRRRPTT — Not Accepted

iii. \( baabbaa \)
   **Solution:** SRRRPPTT — Not Accepted

iv. \( aaa \)
   **Solution:** SQMT — Not Accepted

v. \( abbbb \)
   **Solution:** SQPPPP — Accepted

(b) Describe in words what language is accepted by this automaton or write a regular expression for the language that is accepted by this automaton.

**Solution:** \( aab^* \cup ba^*bb^* \cup aab \cup aabab^* \)

3. (30 Points) Consider the following nondeterministic finite automaton.

(a) Determine if the automaton accepts the following words. If the word is accepted, display the sequence of states that drive the automaton to a favorable state.

i. \( bab \)
   **Solution:** SQSPRPR — Accepted
ii.  $aaa$
   **Solution:** SPSR — Accepted

iii.  $aaaa$
   **Solution:** Not Accepted

iv.  $abab$
   **Solution:** SRQRPR — Accepted

v.  $aaabbbbaaa$
   **Solution:** SPSRQSQSPSR — Accepted

(b) Convert this nondeterministic automaton into an equivalent deterministic automaton.
   **Solution:**

![Diagram of a nondeterministic automaton](diagram1.png)

4. (15 Points) Convert the regular expression

   \[ ba(ba)^*(ab \cup bb)^*a \]

into a finite automaton that accepts this language.
   **Solution:**

![Diagram of a finite automaton](diagram2.png)

5. (20 Points) Convert the following deterministic finite automaton into a regular expression.

   ![Diagram of a deterministic finite automaton](diagram3.png)

   **Solution:**

   \[ (bb \cup ((ba \cup ab)(a(aa \cup b))^{*}(aabb \cup b))^{*}a)^*b. \]

6. (10 Points) Let $\Sigma = \{a, b\}$, write a regular expression representing the language of all strings over $\Sigma$ that do not begin with $bb$ and must end in $aaa$.
   **Solution:**

   \[ (aa \cup ab \cup ba)(a \cup b)^*aaa \cup baaa \cup aaaa \cup aaa. \]

7. (10 Points) Given $\Sigma = \{a, b\}$, show that $L = \{w | bbaab is a substring of w\}$ is a regular language.
   **Solution:**

   \[ (a \cup b)^*bbaab(a \cup b)^*. \]