Definitions and Short Answer: (5 Points Each): Answer all of the following.

1. Define a computation. — Page 3 of the text.
2. Define a Deterministic Finite Automaton. — Page 14 of the text.
3. Define a Nondeterministic Finite Automaton. — Page 22 of the text.
4. What is the primary property of a Finite Automaton, as far as a model of computation is concerned. — Page 14 of the text, it has no auxiliary memory.

DFA Computations: (10 Points): In the automaton diagram below the state labels are below each state (i.e. S, P, Q,...). Do not use the names q0, q1,... to refer to the states. Also note that the loop on state M (the one with both a and b on it) means that there are two loops at M, one labeled a and one labeled b.

1. For each of the following display the sequence of configurations and state if the word is accepted by the language of the automaton, or not.
   (a) abaa
   \textbf{Solution:} (S, abaa) \rightarrow (P, baa) \rightarrow (R, aa) \rightarrow (Q, a) \rightarrow (R, e) — Accepted.
   (b) abbbaaaba
   \textbf{Solution:} (S, abbbaaaba) \rightarrow (P, bbbbaaaba) \rightarrow (R, bbaaaba) \rightarrow (T, baaaba) \rightarrow (T, aaaba) \rightarrow (Q, aaba) \rightarrow (R, aba) \rightarrow (Q, ba) \rightarrow (P, a) \rightarrow (M, e) — Accepted.
   (c) bbbbb
   \textbf{Solution:} (S, bbbbb) \rightarrow (Q, bbbbb) \rightarrow (P, bbb) \rightarrow (R, bb) \rightarrow (T, b) \rightarrow (T, e) — Not Accepted.

2. If we denote the above automaton as A, is \( L(A) \) finite or infinite? Why?
   \textbf{Solution:} Infinite, there is a loop in the automaton between the initial state and a favorable state. Furthermore is also a loop that contains a favorable state.

3. Is \( L(ba(baba)^\ast) \subset L(A) \)? Why or why not?
   \textbf{Solution:} Yes, since ba lands in the favorable state R and bababa lands in the favorable trap state M.

4. Is \( L(ba(ba)^\ast) \subset L(A) \)? Why or why not?
   \textbf{Solution:} No, baba \( \in L(ba(ba)^\ast) \) but baba \( \notin L(A) \).
NFA Computations: (10 Points): In the automaton diagram below the state labels are below each state (i.e., $S, P, Q, ...$). Do not use the names $q_0, q_1, ...$ to refer to the states. Also note that the transition from $R$ to $T$ means that there are two transitions from $R$ to $T$, one labeled $a$ and one labeled $e$.

1. For each of the following, if the word is accepted display a sequence of configurations verifying its acceptance. If it is not accepted simply state that fact.
   
   (a) $abaa$
   
   **Solution:** Not Accepted.

   (b) $babb$
   
   **Solution:** $(S, babb) \rightarrow (Q, abb) \rightarrow (R, abb) \rightarrow (T, bb) \rightarrow (P, b) \rightarrow (Q, e) — Accepted.$

   (c) $ab$
   
   **Solution:** $(S, ab) \rightarrow (P, b) \rightarrow (Q, e) — Accepted.$

   (d) $bbb$
   
   **Solution:** $(S, bbb) \rightarrow (Q, bb) \rightarrow (R, bb) \rightarrow (T, bb) \rightarrow (P, b) \rightarrow (Q, e) — Accepted.$

2. List all words of length 3 or less that do not belong to $L(A)$.
   
   **Solution:** \{a, aa, ba, bb, aaa, aab, aba, baa, bba, bab\}

Proofs: (15 Points Each): Do any (and only) two of the following.

1. Is the following language a regular language over $\Sigma = \{a, b\}$? If so, give its regular expression or a set theoretic proof and if not, prove it. $L = \{w \mid \text{abaab is a substring of } w \text{ and } w \text{ ends with } baaba\}$.
   
   **Solution:** Yes the language is regular, $L = (a \cup b)^*abaab(a \cup b)^*baaba \cup (a \cup b)^*abaaba$

2. Is the following language a regular language over $\Sigma = \{a, b\}$? If so, give its regular expression or a set theoretic proof and if not, prove it. $L = \{w \mid (a^n b)^n \text{ is a substring of } w \text{ and } n > 0\}$.
   
   **Solution:** No this language is not regular. We proceed to prove this by contradiction. Assume that $L$ is regular and let $n$ denote the number $n$ given in the Pumping Lemma. Consider the word $w = (a^n b)^n$. Since $|w| = (n + 1)^n > n$ the Pumping Lemma implies that $w = xyz$ with $|xy| \leq n$. So $y = a^m$ for $1 \leq m \leq n$ and also by the Pumping Lemma $xy^2z = a^{n + m} b(a^n b)^{n-1}$ must be an element of $L$ which is clearly not the case. Hence, $L$ is not regular.

3. Is the language of all strings $w$ over $\Sigma = \{a, b\}$ where every $b$ in $w$ is followed immediately by an $a$ a regular language? If so, give its regular expression or a set theoretic proof and if not, prove it.
   
   **Solution:** Yes the language is regular,

   $L = (a \cup ba)^*$

   Note that we could also verify this by noting that $L = \overline{L'}$ where $L'$ is the language of all words that contain $bb$, i.e. $L' = (a \cup b)^*bb(a \cup b)^*$. Since $L'$ is regular so is $L$. 
4. Is the following language a regular language over $\Sigma = \{a, b\}$? If so, give its regular expression or a set theoretic proof and if not, prove it. $L = \{w \mid w \text{ begins with } ab^nab^n\}$.

**Solution:** No this language is not regular. We proceed to prove this by contradiction. Assume that $L$ is regular and let $n$ denote the number $n$ given in the Pumping Lemma. Consider the word $w = ab^nab^n$. Since $|w| = 2n + 2 > n$ the Pumping Lemma implies that $w = xyz$ with $|xy| \leq n$. So we have that either $y = ab^m$ for $0 \leq m < n$ or $y = b^m$ for $1 \leq m < n$. In the first case, applying the Pumping Lemma again gives $xy^2z = ab^mab^nab^n$ must be an element of $L$ which is clearly not the case since $m < n$. In the second case we have $xz = ab^{n-m}ab^n$ must be an element of $L$ which is clearly not the case since $m \geq 1$. So either case gives a contradiction and hence $L$ is not regular.

**Algorithms: (15 Points Each):** Do any (and only) two of the following.

1. Using the algorithm discussed in class and in the text, convert the automaton in the NFA Computations exercise to a DFA that accepts the same language.

   **Solution:**

   ![Diagram](image)

2. Using the algorithm discussed in class and in the text, convert the following NFA to a regular expression.

   **Solution:**

   $$(ba \cup aa^*b \cup (ba \cup aa^*b)a)a^* = (ba \cup aa^*b)a^*$$

3. Using the algorithm discussed in class and in the text, convert the regular expression

   $$(a(aba \cup ba)^*(a^* \cup bba))^*$$

   to an NFA that accepts the same language.
Solution:

4. Using the algorithms discussed in class and in the text, first create the equivalence classes of indistinguishable states for the following automaton and then create an automaton with a minimum number of states that is equivalent to the given automaton.

Extra Credit: Convert the automaton in the DFA Computations exercise to a regular expression that accepts the same language.

Solution: \[(aa \cup bba)((a \cup b)^* \cup (ab \cup b(bb \cup a))((bb \cup a))\ast(e \cup aba(a \cup b)^\ast) \cup (ab \cup b(bb \cup a))((bb \cup a))\ast b(a(bb \cup a)(a(bb \cup a))\ast b \cup b)^\ast (a(bb \cup a)(bb \cup a))\ast(e \cup aba(a \cup b))\ast) \cup aba(a \cup b)^\ast)\]