1. (15 Points) Consider the following DFA, $A$.

(a) Determine if the automaton accepts the following words. Display the sequence of states for each word.
   i. $baabbba$ — SQTRMMMT — not accepted.
   ii. $aaaaa$ — SPQRT — not accepted.
   iii. $abaabb$ — SPRTRMM — accepted.

(b) Is $L(aba^*b^*) \subset L(A)$? Why or why not? — No, $aba$ is not accepted.

(c) Is $\{b^n a^m \mid n, m > 0$ and $n$ and $m$ are even$\} \subset L(A)$? Why or why not? — Yes, the word must start with $bb$, driving you to $R$. From there any set of an even number of $a$’s will return you to $R$. Any set of $b$’s will take you to $M$ and from there two $a$’s will take you back to $R$.

2. (15 Points) Consider the following NFA, $A$.

(a) Determine if the automaton accepts the following words. If it does, display the sequence of states that drive the word to an acceptable state.
   i. $aababb$ — accepted — SPTMFRSQFRSFRSQF
   ii. $aabaaa$ — accepted — SPTMFQFRF
   iii. $baba$ — accepted — SRQMF
   iv. $aaaaaa$ — not accepted.

(b) Is $L(baaa(ba)^*) \subset L(A)$? Why or why not? Yes, $baaa$ drives the automaton to $F$ via SRQTMF then from there $ba$ drives you back to $F$ by RQF. In any case you end on a favorable state.
3. (20 Points) Do one (and only one) of the following,

(a) Convert the following NFA to a DFA,

Solution:

(b) Convert the regular expression \((aa^* \cup b^*ab)a^*aabb^*(e \cup a \cup aaa)\) to an NFA,

Solution:
4. (20 Points) Convert the following NFA to a regular expression,

\[ ab^*ba^*a \cup (ab^*ba^*a \cup e)(b \cup (b \cup ab^*b)a^*a)(a \cup (b \cup ab^*b)a^*a) \]

Solution: \( ab^*ba^*a \cup (ab^*ba^*a \cup e)(b \cup (b \cup ab^*b)a^*a)(a \cup (b \cup ab^*b)a^*a) \)

5. (20 Points) Minimize the number of states for the following DFA,

Solution: The equivalence class chart and the converted automaton are

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6. (20 Points) Prove that the language \( L = \{ a^n b^m | n, m > 0, n \text{ is even and } t = n/2 \} \) is not regular. Make sure you verify all statements completely.

Solution: Assume that \( L \) is a regular and let \( n \) be the value from the pumping lemma. Let \( w = a^n b^{2n} \), then \( w = xyz \) with \( |xy| \leq n \) and \( y \) non-empty. Thus, \( xy = a^k \) for some \( 1 \leq k \leq n \) and so \( y = a^p \) for some \( 1 \leq p \leq n \). But \( xy^2z = a^{n+p}b^{2n} \notin L \), which contradicts \( L \) being regular.