1. (15 Points) Consider the following DFA, $A$.

\[ A \]

(a) Determine if the automaton accepts the following words. Display the sequence of states for each word.

i. $aabbab$ — Not Accepted — SPRTRMT
ii. $bababa$ — Not Accepted — SQRTWWM
iii. $abaabb$ — Accepted — SPTWMTR

(b) Is $\{(aa)^t(bb)^r \mid t, r > 0\} \subseteq L(A)$? Why or why not? — Yes, $aa$ takes you to $R$ and any more $a$’s takes you to $M$. From $R$ each $bb$ returns you to $R$ and from $M$ a $bb$ takes you to $R$. Hence, $\{(aa)^t(bb)^r \mid t, r > 0\} \subseteq L(A)$.

(c) Let $L = L(A)$ and let $M = \{w \mid$ the number of $a$’s equals the number of $b$’s$\}$, what is the shortest word in $L \cap M$? — $ba$

2. (15 Points) Consider the following NFA, $A$.

\[ A \]

(a) Determine if the automaton accepts the following words. If it does, display the sequence of states that drive the word to an acceptable state.

i. $bbaaab$ — Accepted — SPTFQFQTF
ii. $ababab$ — Accepted — SRFMTFQTF
iii. $aabbaa$ — Not Accepted

(b) If $w \in L(A)$ and $w$ has an even number of $a$’s must it also have an even number of $b$’s? Why or why not? — No, $aab \in L(A)$

(c) If $w \in L(A)$ and $w$ has an odd number of $a$’s must it also have an odd number of $b$’s? Why or why not? — No, $a \in L(A)$
3. (20 Points) Do one (and only one) of the following,

(a) Convert the following NFA to a DFA,

Solution:

(b) Convert the regular expression \((aba^* \cup bab^*)^*ab(a \cup b^*a)\) to an NFA,

Solution:
4. (20 Points) Convert the following NFA to a regular expression,

\[ a^*ab \cup (a^*a \cup b)(b \cup aa^*)^* (aa^*ab \cup b) \]

Solution: 

5. (20 Points) Minimize the number of states for the the following DFA,

Solution: The equivalence class chart is as follows

Since the states have all split all of the states are distinguishable so the automaton already has the minimum number of states.

6. (20 Points) Prove that the language \( L = \{ a^t b^3 t \mid t > 0 \} \) is not regular. Make sure you verify all statements completely.

Solution: By way of contradiction assume that \( L \) is a regular language. By the pumping lemma there is a fixed number \( n > 0 \) such that for any word \( w \in L \) with \( |w| \geq n \) we can write \( w = xyz \) with \( y \) nonempty, \( |xy| \leq n \) and \( xy^iz \in L \) for any integer \( i \geq 0 \). Let \( w = a^nb^{3n} \), since \( |w| = 4n > n \) we can write \( w = xyz \) with \( |xy| \leq n \). Hence \( xy = a^r \) for some \( 0 < r \leq n \) and since \( y \) is not empty we must have \( y = a^p \) with \( 0 < p \leq n \). Using the final condition from the pumping lemma we must have \( xy^2z \in L \), but \( xy^2z = a^{n+p}b^{3n} \) which is not in \( L \) since \( 3(n + p) = 3n + 3p > 3n \). This contradicts the pumping lemma and hence our assumption that \( L \) was not regular. Therefore \( L \) is regular.