Chapter 6 – Interest Rates and Bond Valuation

Definition and Description of Bonds

Long-term debt-loosely, bonds with a maturity of one year or more
Short-term debt-less than a year to maturity, also called unfunded debt
Bond-strictly speaking, secured debt; but used to describe all long-term debt

Bond Valuation

Terminology (Symbols)

A. **Par Value** (Face Value): contractually set (multiples of $1000 denominations)
   1. paid back as the terminal cash flow (principle)
   2. symbols \( F, FV, \) and \( M \) often used
B. **Coupon Rate**: contractually set; used to determine the periodic interest (coupon) payments
   1. symbols \( CR \) (coupon rate)
C. **Coupon Payment**: the periodic interest payment (function of face value and coupon rate)
   1. symbols \( PMT, CP \) (coupon payment), or \( INT \) (interest) used
   2. \( CP = (Par)(CR) \)
D. **Market Rate**: the current “going rate” for bonds of similar risk and term
   1. rate used for all TVM calculations
   2. “Yield to Maturity” (YTM) when calculated
   3. symbols \( r_d \) or \( k_d \) (rate or cost of debt) often used
E. **Term**: number of periods until the bond matures and principle returned
   1. must be adjusted for compounding periods per year
   2. symbol \( n \) often used

Formula

A. Recognize that the cash flow stream is an annuity with a set payment of the coupon payments (CP) and a lump sum in the amount of the principle (Par)
B. Recognize that the present value (price) of any cash flow stream is the sum of the present values of the parts of the cash flow stream (in this case the annuity component and the lump sum component)

\[
PRICE = PV = \sum_{t=1}^{n} \frac{CF_t}{(1+k_d)^t} = CP \sum_{t=1}^{n} \frac{1}{(1+k_d)^t} + \frac{Par}{(1+k_d)^n}
\]

1. \( CP \) and \( Par \) will be given (contractually set)
2. Given market rate \( k_d \) you can solve for \( PRICE \)
3. Given \( PRICE \) you can solve for \( k_d \) (market rate or yield to maturity)
Sample Problem #1 – Solving for Price

A. Given a 4-year bond with a $1000 face value and a 5% coupon rate, annual compounding (annual periodic interest payments), find the price of the bond if the market rate for similar bonds is 6%.

B. Numerical Solution
   1. Step 1: Calculate the coupon payment
      \[ CP = Par(CR) = 1000 \times 0.05 = 50 \]
   2. Step 2: Assign values to the variables
   3. Step 3: Apply the formula
      \[ PRICE = PV = 50 \sum_{t=1}^{4} \frac{1}{(1.06)^t} + \frac{1,000}{(1.06)^4} = \$965.35 \]

C. Tabular Solution
   1. Step 1 & Step 2: Same as above
   2. Step 3: Determine table values for both the annuity and lump sum (enter the tables with n and \( k_a \))
   3. Step 4: Apply the formula
      \[ Price = PV = V_B = 50(3.4651) + 1,000(0.7921) = \$965.355 \]

D. Calculator solution
   1. Step 1 & Step 2: Same as above
   2. Step 3: Input variables into calculator
   3. Step 4: Compute price
      
      \[
      \begin{array}{l}
      \text{N} \\
      \text{I} \\
      \text{Cpt PV} \\
      \text{Pmt} \\
      \text{FV}
      \end{array}
      \]
Sample Problem #2 – Solving for Yield to Maturity Given Price

A. Assume now that you are given a quote on the price for the bond in Problem #1 and it is selling for $932.25. What is the yield to maturity (market rate for debt) for the bond?

B. Numerical and Tabular Solutions require trial and error to solve.

C. Calculator Solution
   1. Step 1: Assign values to the variables
   2. Step 2: Input variables into calculator
   3. Compute rd

V: Bond Pricing Principles

A. These five simple rules will allow you to better understand the relationship between PRICE and yield (kd) for a given bond (inverse relationship)
   1. When coupon rate = kd the bond sells at par value (PRICE = Par)
   2. When coupon rate > kd the bond sells at a premium (PRICE > Par)
   3. When coupon rate < kd the bond sells at a discount (PRICE < Par)
   4. At maturity PRICE = Par for all values of kd
   5. The longer the time to maturity the greater the premium (discount)

B. Discuss the graph below
Bond fact #1:

There is an inverse relationship between interest rates and bond prices. If interest rates increase, bond prices decrease. If interest rates decrease, bond prices increase.

Bond fact #2:

Interest rate risk is the risk that if interest rates increase, bond prices will decrease. All else the same, a longer term bond will have more interest rate risk than a shorter term bond.

Bond fact #3:

All else the same, there is an inverse relationship between the coupon rate and interest rate risk. A bond with a lower coupon has more interest rate risk than a bond with a higher coupon.
Zero coupon bonds

Suppose we have the following bond:

Par = $1,000
Coupon rate = 0%
Maturity = 15 years
YTM = 9%

What is the price of the bond?

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>Cpt PV</th>
<th>Pmt</th>
<th>FV</th>
</tr>
</thead>
</table>

What is the price of the bond if we use semi-annual compounding?

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>Cpt PV</th>
<th>Pmt</th>
<th>FV</th>
</tr>
</thead>
</table>
Read Section 6.2
Read Section 6.3
Read Section 6.4
Read Section 6.5
Read Section 6.6

Term Structure of Interest Rates